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Fault-Tolerant H_∞ Control of Active Suspension Systems with Time Delay

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Abstract. In this study, we address the problem of fault-tolerant control (FTC) H_∞ of active vehicle suspension systems (ASS) systems with time delays and actuator fault. The H_∞ performance criterion is employed to assess ride comfort, allowing for the consideration of general road disturbances. A method for designing the controller established on Linear Matrix Inequalities (LMIs) has been suggested to handle actuator faults represented by a scaling factor. The control design guarantees the asymptotic stability of the system, mitigates the effects of actuator malfunctions, and maintains H_∞ performance although in presence of defective actuators. The effecency and robustness of the proposed approach are demonstrated through simulations and comparative analysis.

Introduction

Modern vehicles are significantly more comfortable and secure compared to their predecessors. The suspension system (SS) absorbs shocks from forces acting betwixt the road and the vehicle chassis, ensuring a smooth ride and reliable handling performance. SS are generally categorized into three classes: passive, semi-active and active suspensios (AS) [1,2]. The main goal of research on (ASS) is to improve suspension performance by optimizing ride comfort, suspension deflection and road handling [3].

Significant attention has been given to controlling ASS in recent years, leading to numerous notable advancements [4]. Researchers have proposed various approaches to optimise SS performance by employing a range of control techniques, including adaptive control [5], H_∞ control [6].

Time delays are common in various fields of physical systems and can lead to instability and reduced control performance. Increasing methods have been developed to mitigate the effects of delays across different types of systems, two-dimensional systems [7–8] and fuzzy systems [9,10] among others. For vehicle SS specifically, noteworthy contributions include [11] and [12].

Designing FTC systems, even in the presence of faults, is a difficult problem for ASS because of the unpredictability and inevitability of defects. These unexpected malfunctions may a source of system failure and a noticeable decline in his performance. Because of these difficulties, it is essential to design an FTC system capable of automatically compensating for changes in system

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dynamics caused by failures. The term failure tolerance (FT) make reference to the system's potential to continue functioning correctly even if some of its components fail. The capability to sustain functionality under component failures is often referred to as progressive degradation. Specifically, FTC is responsible for ensuring that performance is acceptable and system stability is maintained, even if faults occur. Using different engineering controls, there have been several proposals for FTC approaches [13]. As far as we know, there has been limited research has been conducted on FTC for ASS that account for both actuator delays and failures. This gap motivates and underpins the efforts presented in this article.

This paper presents the development of H_∞ FTC design methods based on a state feedback control law, explicitly considering input time delays and actuator failures. The proposed approach utilises LMI to ensure that the ASS remains asymptotically stable while achieving a predefined level of disturbance attenuation. It also compensates for actuator failures to maintain performance. By presenting simulation results, the proposed method's effectiveness and robustness can be verified.

Materials and Methods

The parameters of a quarter car model depicted in Figure 1 include m_s representing the sprung mass, m_u representing the unsprung mass, c_s represent the damping of SS, k_s represent the stiffness of SS, $u_a^f(t)$ represents the FTC of the ASS, k_t represent the compressibility of the tyre, c_t the damping of the tyre, the displacements of z_s correspond to the sprung displacements and z_u corresponds to the unsprung displacements, z_r represents the road displacement . d: the control input delay assumed to be unknown and constant with a known upper bound($0 \le d \le \bar{d}$).

According to Figure 1, the following is how dynamic equations can be obtained:

$$m_s\ddot{z}_s(t) + c_s[\dot{z}_s(t) - \dot{z}_u(t)] + k_s[z_s(t) - z_u(t)] = u_a^f(t-d),$$
$$m_u\ddot{z}_u(t) + c_s[\dot{z}_u(t) - \dot{z}_s(t)] + k_s[z_u(t) - z_s(t)] + c_t[\dot{z}_u(t) - \dot{z}_r(t)] + k_t[z_u(t) - z_r(t)] = u_a^f(t-d), \quad (1)$$

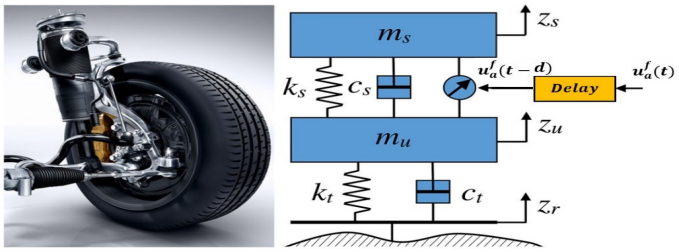

Figure 1. Quarter-car model with an ASS.

The system states are introduced as follows:

$$x_1(t) = z_s(t) - z_u(t), \quad x_2(t) = z_u(t) - z_r(t), \quad x_3(t) = \dot{z}_s(t), \quad x_4(t) = \dot{z}_u(t), \quad (2)$$

The deflection of the suspension, the deflection of the tire, the speed of the sprung mass, and the speed of the unsprung mass are all represented by $x_1(t)$, $x_2(t)$, $x_3(t)$, and $x_4(t)$ respectively. $w(t) = \dot{z}_r(t)$ represents the inputs of disturbance. Let define $x(t) = [x_1(t) \;\; x_2(t) \;\; x_3(t) \;\; x_4(t)]^T$. According to (1), it is possible to rewrite the dynamic equations below:

$$\dot{x}(t) = Rx(t) + Tw(t) + Su_a^f(t-d), \quad (3)$$

where

$$R = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{(c_s+c_t)}{m_u} \end{bmatrix}, \quad T = [0 \quad -1 \quad 0 \quad \tfrac{c_s}{m_u}]^T, \quad S = \left[0 \quad \tfrac{1}{m_s} \quad 0 \quad -\tfrac{1}{m_u}\right]^T,$$

The condition translate into a limitation on the tire force induced by road irregularities, which must remain lower than the total static load supported by the vehicle, namely.

$$k_t[z_u(t) - z_r(t)] < (m_s + m_u)g.$$

Moreover, inherent mechanical constraints of the suspension assembly impose strict bounds on the relative displacement between the unsprung and sprung masses, ensuring that the suspension deflection does not exceed its admissible range $|z_s(t) - z_u(t)| \le z_{max}$, (4)

where z_{max} corresponds to the upper bound of the suspension deflection.

Due to its power constraints, the hydraulic actuator constitutes a significant limitation on the AS system. $|u_a(t)| \le u_{max}$, (5)

where u_{max} indicates the maximum amount of suspension displacement that is allowed..

The controller is designed to operate at the following output quantities:

$$z_1(t) = \ddot{z}_s(t) \qquad z_2(t) = \left[\frac{z_s(t)-z_u(t)}{z_{max}} \;\; \frac{k_t(z_u(t)-z_r(t))}{(m_s+m_u)g}\right]^T \tag{6}$$

Next, we can describe the ASS of the quarter-car can be expressed as:

$$\begin{cases} \dot{x}(t) = Rx(t) + Tw(t) + Su_a^f(t-d) \\ z_1(t) = Mx(t) + Nu_a^f(t-d) \\ z_2(t) = Lx(t) \end{cases} \tag{7}$$

where R, T, and S are defined in (3), and

$$M = \left[-\frac{k_s}{m_s} \quad 0 \quad -\frac{c_s}{m_s} \quad \frac{c_s}{m_s}\right],\ N = \frac{1}{m_s},\qquad L = \begin{bmatrix} \frac{1}{z_{max}} & 0 & 0 & 0 \\ 0 & \frac{k_t}{(m_s+m_u)g} & 0 & 0 \end{bmatrix}.$$

The control equation is formulated as a delayed state feedback mechanism and is expressed as:

$$u_a(t-d) = K_u x(t-d), \tag{8}$$

The state feedback gain matrix, determined through control design procedures, is denoted as: K_u

The operator's fault behavior is modeled as follows.

$$u_a^f(t-d) = Fu_a(t-d), \tag{9}$$

where F stands for the actuator fault matrix, expressed as: $F = diag(f_n),\ 0 \le \check{f}_{ln} \le f_n \le \hat{f}_{un}$, $f_n = 1$, indicate the operation of the actuator under normal conditions, $f_n = 0$ refers to complete loss of actuator control, whereas intermediate values describe partial loss scenarios $0 \le \check{f}_{ln} \le f_n \le \hat{f}_{un},\ f_n \ne 1,$

Substituting Eqs (8) and (9) into Eq (7), the resulting closed-loop dynamics of the system can be expressed as follows:

$$\begin{cases} \dot{x}(t) = Rx(t) + SFK_u x(t-d) + Tw(t) \\ z_1(t) = Mx(t) + NFK_u x(t-d) \\ z_2(t) = Lx(t) \end{cases} \tag{10}$$

Consider the set of disturbances w(t) ∈ L₂[0,∞), such that $||w||_2^2 \le w_{max} < \infty$. The control synthesis problem involves determining a gain K_u that guarantees the following properties[13]: Internal Stability: The equilibrium point of the unforced system $w(t) = 0$ is asymptotically stable. Disturbance Rejection: For all non-zero, the performance inequality $||z_1||_2 < \gamma||w||_2$ holds for a given $w(t) \in L_2[0,\infty)$, where $\gamma > 0$. Constraint Satisfaction: Actuator and output signals remain within specified admissible regions:

$$|\{z_2(t)\}_i| \le 1, \quad i = 1,2, \qquad |u_a(t)| \le u_{max} \tag{11}$$

Remark 1. The fault model (9) takes into account inaccurate actuator outputs. It encompasses the fault model [14] in which sensor and actuator faults are modelled discretely. For simplicity, we introduce the following notations

$$F_0 = f_{01}, J = j_1, E = e_1\ \ |E| = |e_1|,\ f_{01} = \frac{1}{2}(\hat{f}_{u1} + \check{f}_{l1}), j_1 = \frac{\hat{f}_{u1}-\check{f}_{l1}}{\hat{f}_{u1}+\check{f}_{l1}},\ e_1 = \frac{f_1-f_{01}}{f_{01}}, \tag{12}$$

By equations (12) we have $F = F_0(I + E), \in \mathbb{R}$ $|E| \le J \le I,$ (13)

The following lemmas are presented to establish the stability and performance guarantees detailed in the following analysis.

Lemma 1 [15] Let X, Y be matrices of appropriate dimensions and for any scalar $\varepsilon > 0$, the following holds

$$X^T Y + Y^T X \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y$$

Lemma 2 [15] Let R_1 and R_2, be matrices of compatible dimensions. Then, the following inequality is satisfied:

$$R_1 \Sigma R_2 + R_2^T \Sigma R_1^T \leq \beta R_1 U R_1^T + \beta^{-1} R_2^T U R_2$$

where $\beta > 0$, is a positive scalar, and $\Sigma = diag(\sigma_1, \sigma_2, \ldots \sigma_l)$ represents a positive definite diagonal matrix such that $\Sigma \leq U$.

Lemma 3 [16] Assume $a(.) \in \mathbb{R}^{n_a}$, $b(.) \in \mathbb{R}^{n_b}$ and $H \in \mathbb{R}^{n_a \times n_a}$ are defined on the interval Ω. Consequently, for any matrix X,
the interval Ω. Then for any matrices $X \in \mathbb{R}^{n_a}$, $Y \in \mathbb{R}^{n_b}$ and $Z \in \mathbb{R}^{n_a \times n_a}$ the following holds:

$$-2\int_\Omega a^T(\alpha) H b(\alpha) d\alpha \leq \int_\Omega \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y - H \\ * & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} d\alpha, \quad \text{where} \quad \begin{bmatrix} X & Y \\ * & Z \end{bmatrix} > 0.$$

Controller design

In this section, our objective is to design an H_∞ FTC controller capable of stabilizing the vertical movement of the vehicle chassis in the presence of delay and actuator failures.

Theorem 1

Consider the SS (7). For a scalar $\bar{d} > 0$, the closed-loop ASS (10) is asymptotically stable and guarantees an H_∞ performance level γ for all constant delays d such that $0 \leq d \leq \bar{d}$, under the existence of matrices $\tilde{P} > 0$, $\tilde{Z} > 0$, $\tilde{Q} > 0$, $\tilde{X}$ and H satisfying the LMIs (14)-(17). In addition, the resulting H_∞ state-feedback controller is expressed as $u(t) = V \tilde{P}^{-1} x(t)$:

$$\begin{bmatrix} \phi_{11} & \phi_{12} & T & \phi_{14} & \phi_{15} & 0 \\ * & -\tilde{Q} & 0 & \bar{d} V^T F_0^T S^T & V^T F_0^T N^T & V^T J^{\frac{1}{2}} \\ * & * & -\gamma^2 & \bar{d} T^T & 0 & 0 \\ * & * & * & \phi_{44} & \alpha \bar{d} S F_0 J F_0^T N^T & 0 \\ * & * & * & * & \phi_{55} & 0 \\ * & * & * & * & * & -\alpha I \end{bmatrix} < 0 \tag{14}$$

$$\begin{bmatrix} \tilde{X} & H \\ * & \tilde{P} \tilde{Z}^{-1} \tilde{P} \end{bmatrix} > 0 \tag{15}$$

$$\begin{bmatrix} -I + \varepsilon F_0 J F_0^T & \sqrt{\rho} F_0 V & 0 \\ * & -u_{max}^2 \tilde{P} & \sqrt{J\rho} V^T \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} -I & \sqrt{\rho}\{L\}_i \tilde{P} \\ * & -\tilde{P} \end{bmatrix} < 0, \quad i = 1,2. \tag{17}$$

where

$\phi_{11} = \tilde{P} R^T + R \tilde{P} + \bar{d} \tilde{X} + H + H^T + \tilde{Q} + \alpha S F_0 J F_0^T S^T$, $\phi_{12} = S F_0 V - H$, $\phi_{14} = \bar{d} \tilde{P} R^T + \alpha \bar{d} S F_0 J F_0^T S^T$, $\phi_{44} = -\bar{d} \tilde{Z} + \alpha \bar{d}^2 S F_0 J F_0^T S^T$, $\phi_{15} = \tilde{P} M^T + \alpha S F_0 J F_0^T S^T$, $\phi_{55} = -I + \alpha N F_0 J F_0^T N^T$, $V = K_u \tilde{P}$

Proof. According to the well-known Leibniz-Newton formula and equations (8), then we can write:

$$x(t - d) = x(t) - \int_{t-d}^{t} \dot{x}(s) ds \tag{18}$$

Then substituting $x(t - d)$ into equation (10) gives

$$\dot{x}(t) = (R + S F K_u) x(t) - S F K_u \int_{t-d}^{t} \dot{x}(s) ds + T w(t) \tag{19}$$

Choosing a Lyapunov functional condidate $V(t)$ as

$$V(t) = V_1 + V_2 + V_3, \tag{20}$$

With

$$V_1 = x^T(t)Px(t),\ V_2 = \int_{-d}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha d\beta,\ V_3 \int_{t-d}^{t} x^T(\alpha)Qx(\alpha)d\alpha \tag{21}$$

and $P = P^T > 0,\ Z = Z^T > 0$ and $Q = Q^T > 0$ are matrices to be chosen.

Along the trajectory of system (18), the derivative of V is expressed as

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \leq x^T(R^TP + PR + \bar{d}X + Y^T + Y)x(t) + 2\,x^T(t)(PSFK_u - Y)x(t-d) + \bar{d}[Rx(t) + Tw(t) + SFK_ux(t-d)]^TZ[Rx(t) + Tw(t) + SFK_ux(t-d)] + x^T(t)Qx(t) - x^T(t-d)Qx(t-d)) + w^T(t)T^TPx(t) + x^T(t)PTw(t). \tag{22}$$

We consider zero initial conditions i.e. $\phi(t) = 0,\quad \forall t \in [-d, 0]$,

Define $$J_{zw} = \int_0^{\infty}[z_1^T(t)z_1(t) - \gamma^2 w^T(t)w(t)]\,dt \tag{23}$$

then,

$$\Psi = \begin{bmatrix} \Upsilon & PSFK_u - Y + \bar{d}R^TZSFK_u + M^TNFK_u & \bar{d}R^TZT + PT \\ * & \bar{d}(SFK_u)^TZ(SFK_u) - Q + (NFK_u)^T(NFK_u) & \bar{d}(SFK_u)^TZT \\ * & * & -\gamma^2 I + \bar{d}T^TZT \end{bmatrix} < 0, \tag{24}$$

Where $$\Upsilon = R^TP + PR + \bar{d}X + Y^T + Y + Q + \bar{d}R^TZR + M^TM,$$

When we suppose the input zero disturbance, that is to say that $w(t) = 0$, if Eq. (24) holds, then $\dot{V}(x(t)) < 0$ i.e. the asymptotic stability of ASS (10) is thus proven. Using the Schur complement to Ψ, substituting F with its expression given in (13), and invoking Lemma 2, inequality (24) can be given as

$$\begin{bmatrix} \Xi & SF_0V - H & T & \bar{d}\tilde{P}R^T & \tilde{P}M^T \\ * & -\tilde{Q} & 0 & \bar{d}\,V^TF_0^TS^T & V^TF_0^TN^T \\ * & * & -\gamma^2 I & \bar{d}T^T & 0 \\ * & * & * & -\bar{d}\tilde{Z} & 0 \\ * & * & * & * & -I \end{bmatrix} + \alpha \begin{bmatrix} SF_0 \\ 0 \\ 0 \\ \bar{d}SF_0 \\ \bar{d}SF_0 \end{bmatrix} J \begin{bmatrix} SF_0 \\ 0 \\ 0 \\ \bar{d}SF_0 \\ \bar{d}SF_0 \end{bmatrix}^T + \alpha^{-1} \begin{bmatrix} 0 \\ V^T \\ 0 \\ 0 \\ 0 \end{bmatrix} J \begin{bmatrix} 0 \\ V^T \\ 0 \\ 0 \\ 0 \end{bmatrix}^T < 0 \tag{25}$$

Where

$\Xi = \tilde{P}R^T + R\tilde{P} + \bar{d}\tilde{X} + H^T + H + \tilde{Q}, \tilde{P} = P^{-1}, V = K_u\tilde{P},\ \tilde{X} = \tilde{P}X\tilde{P},\ H = \tilde{P}Y\tilde{P},\ \tilde{Q} = \tilde{P}Q\tilde{P},$
$\tilde{Z} = Z^{-1}$

By using lemma 2 we can obtain LMI (14).

Following a similar approach, we perform pre- and post-multiplication of the second inequality in Lemma 3 using $diag(\tilde{P},\ \tilde{P})^T$ and its transpose, and subsequently carrying out the substitution, one obtains $\tilde{X} = \tilde{P}X\tilde{P},\ H = \tilde{P}Y\tilde{P},$ and $\tilde{Z} = Z^{-1}$, then LMI (15) hold.

We have when $w(t) = 0$, from $\dot{V}(x(t))$ and Eq. (23) we can write

$$\dot{V}(t) + z_1^T(t)z_1(t) - \gamma^2 w^T(t)w(t) < 0, \tag{26}$$

Consequently, the ASS is asymptotically stable, which establishes the desired performance. In the following, we demonstrate that the hard constraints are satisfied. Inequality (26) ensures that $\dot{V}(t) - \gamma^2 w^T(t)w(t) < 0$. By integrating this inequality over the interval [0,t], for any $t > 0$, we obtain:

$$V(t) - V(0) < \gamma^2 \int_0^t w^T(t)w(t)dt < \gamma^2||w||_2^2 \tag{27}$$

Taking into account that the second term of the Lyapunov functional (20) is positive, we have $x^T(t)Px(t) < \rho$, with $\rho = \gamma^2 w_{max} + V(0)$.

$$\max_{t>0}|\{z_2(t)\}_i|^2 = \max_{t>0}\|x^T(t)\{L\}_i^T\{L\}_ix(t)\|_2 = \max_{t>0} < \rho\theta_{max}(P^{-\frac{1}{2}}\{L\}_i^T\{L\}_iP^{-\frac{1}{2}}),\quad i = 1,2.$$

$$\max_{t>0}\left|u_a^f(t)\right|^2 = \max_{t>0}\|FK_ux(t)\|_2^2 = \max_{t>0}\left\|x^T(t)K_u^TF^TFK_ux(t)\right\|_2 < \rho\theta_{max}\left(P^{-\frac{1}{2}}K_u^TF^TFK_uP^{-\frac{1}{2}}\right).$$

Let θ_{max} denote the maximal eigenvalue. Then, from the inequalities above, the constraints in (11) are ensured if

$$\rho\left(P^{-\frac{1}{2}}\{L\}_i^T\{L\}_i P^{-\frac{1}{2}}\right) < I,\ i = 1,2. \tag{28}$$

$$\rho\left(P^{-\frac{1}{2}}K_u^T F^T F K_u P^{-\frac{1}{2}}\right) < u_{max}^2 I. \tag{29}$$

by Schur complement to (28) and (29) and by substituting F with its expression and making use of Lemma 2, then we can obtain then (16) and (17) can be obtained.

Results and Discussion

The effectiveness of the proposed control approach was verified through a detailed simulation of the quarter-car SS, based on the parameters reported in Table 1.

Table 1. Parameters of the Quarter-Car SS Model

m_u	m_s	k_t	k_s	c_t	c_s
114kg	973kg	101115N/m	42720N/m	1095Ns/m	14.6Ns/m

Assuming a maximum allowable suspension travel of $z_{max} = 0.08m$ and a maximum output power of $u_{max} = 1500$, we choose $\rho = 1$. By fixing the actuator failure matrix $F = I$ in normal actuator operation, and setting $d = 9.2ms$, we can solve the linear inequality matrix (LMI) in Theorem 1 to obtain the possible solution. Table 2 shows the corresponding lower boundary of the H_∞ performance corresponding to different upper limits of of d.

Table 2. The minimum of H_∞ performance index

$d(ms)$		0.85	0.9	1	1.5
$min\gamma$		2.2131	2.2356	2.2358	2.2360

The obtained gain matrices for the ASS, corresponding to the cases with and without actuator failure, are listed consecutively as:

$\boldsymbol{K_{uf} = 10^3 \times [6.4220 \quad -1.3655 \quad -5.4775 \quad 0.6937]}$ $\boldsymbol{K_{uwf} = 10^3 \times [8.7664 \quad -0.9297 \quad -8.4466 \quad 1.0701]}$.

For comparison, we consider the AS on a quarter of the vehicle in reference [17], where this paper discusses actuator failures. Our results, presented in Table 3, are clearly better than those in reference [17]. Therefore, we can say that our results are less conservative than those of the aforementioned paper.

Table 3. Comparing performance γ

	$H_\infty\ performance\ index\ \gamma_{min}$
Ref [17]	18.3676
Theorem 1	1.5626

Evaluation for bump response

To investigate the suspension behavior with respect to the aforementioned performance criteria, considering an isolated bump on a smooth road profile is considered. The disturbance input corresponding to this case is defined as:

$$z_r(t) = \begin{cases} \frac{A}{2}(1 - \cos(\frac{2\pi V}{h}) & if\ 0 \le t \le \frac{h}{V} \\ 0 & if\ 0 > \frac{h}{V} \end{cases} \tag{30}$$

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Where h and A are the length and height of the bump, respectively, with $A = 60\,mm$, assumed to be. It is assumed $A = 60\,mm$, $h = 5\,m$ and the vehicle longitudinal velocity $V = 30\,km/h, d = 6ms$.

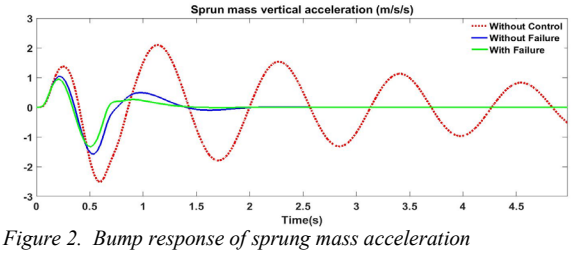

Figure 2. Bump response of sprung mass acceleration

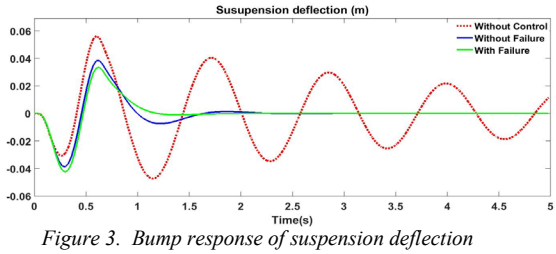

Figure 3. Bump response of suspension deflection

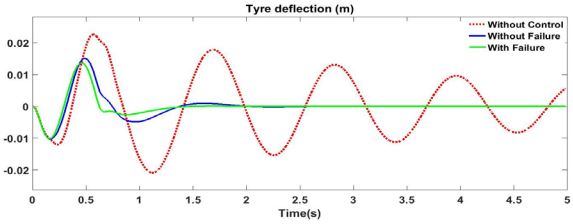

Figure 4. Bump response of tyre deflection

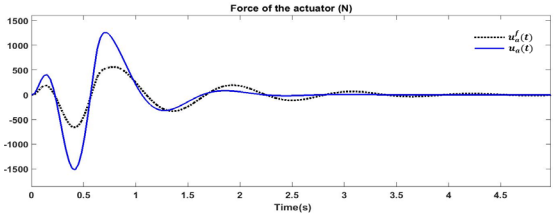

Figure 5. Force of the actuator

Simulation results presented in Figures 2-4 illustrate the resulting performance of the ASS using the proposed FTC strategy. under the proposed FTC controller. These results confirm that, despite delays and actuator failures, the controller provides superior road handling ride comfort, while maintaining acceptable suspension deflection and actuator forces remain within safe limits.

Evolution of Random response

In the time domain, a non-stationary random road profile is defined as

$$\dot{z}_r(t) + 2\pi u n_0 z_r(t) = \sqrt{G_q(n_0) v w(t)} \tag{31}$$

where $w(t)$ is a white noise in the time-domain, v a constant velocity $v = 20m/s$, n_0 is

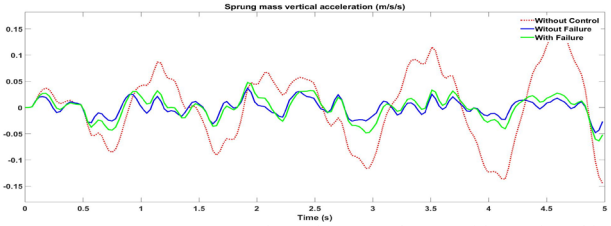

Figure 6. Sprung mass vertical accelerations for variable road profile

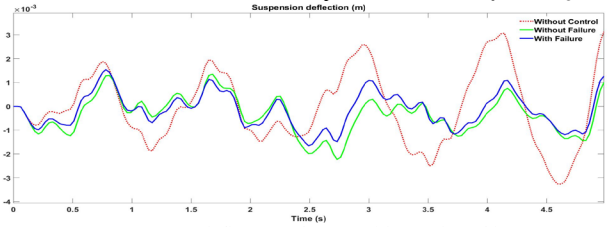

Figure 7. Suspension deflections for variable road profile

The spatial reference frequency is defined as $n_0 = 0.1(cycle/m)$, and $G_q(n_0)$ is the road roughness coefficient. In accordance with ISO-2631, the average road roughness rating is shown here: $G_q(n_0) = 64 \times 10^{-6} m^3, d = 6ms$.

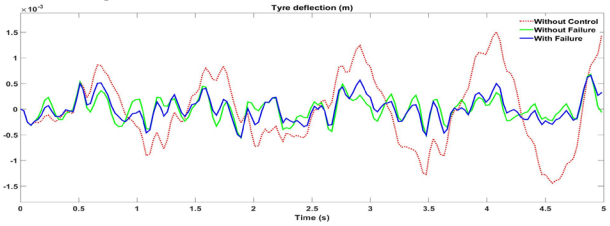

Figure 8. Tyre deflections for variable road profile

Figures 6-8 illustrate the evolution of the system's main parameters when actuator failures and time delays are taken into account. These figures clearly show that the H_∞ control strategy achieves better performance than the passive SS. The simulation results verify the effectiveness of the designed FTC scheme.

Conclusion

This study has addressed the FTC problem for ASS by explicitly considering actuator faults and input time delays. An H_∞ based FTC strategy has been developed to ensure that the ASS remains asymptotically stable while achieving the desired level of ride comfort and road handling performance. The controller synthesis relies on LMI-based conditions that guarantee robustness against external disturbances and actuator degradations. The developed strategy has been validated via simulations conducted on a quarter-car ASS model, and simulation results demonstrate its ability to preserve stability and performance under actuator faults, time delays, road excitations, and varying operating conditions. Furthermore, the proposed framework can be readily extended to more complex vehicle configurations.

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