Plastic flow modelling under abrupt strain-path change and parameter identification method

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Abstract. Rational plastic flow modelling under abrupt strain-path change is crucial for accurate metal forming simulations. Previous experimental studies have revealed that many metals exhibit "corner effects" after abrupt strain-path change in the plastic stage, including a sharp vertex produced on the subsequent yield surface and apparent non-normality of the plastic flow direction. Some advanced models can give reasonable predictions on the above phenomenon, but they have not been widely used in metal forming simulations due to their complexity. A simple non-associated flow rule is developed here to describe corner effects. A stable stress update algorithm is developed for this rule based on the fully implicit backward Euler method, and its validation is verified by FE simulations of the nonlinear tension-torsion loading experiments. In order to determine the material parameters of these non-associated flow rules representing corner effects, a general identification method is proposed based on the equivalent tangential shear modulus reduction. The results show that, compared with the previous theories, the present rule performs better in predicting corner effects. In addition, although the apparent tangential shear modulus reduction is observed during the nonlinear experiments, the modulus does not seem to decrease to near zero.

Introduction

Reliable metal forming simulations put forward a higher request for the plastic flow modelling of complex mechanical responses, such as corner effects. Previous experimental studies [1-4] have revealed that many metals exhibit corner effects after abrupt strain-path change in the plastic stage, including a sharp vertex produced on the subsequent yield surface and apparent non-normality of the plastic flow direction. Corner effects are concomitant with the apparent decrease of shear modulus, thus playing an important role in the problems of strain localization, e.g., plastic buckling [5,6], shear band formation [7,8], necking forming limit [9,10].

The existence of vertexes on the static yield surface is still controversial because it may be caused by the visco-plasticity of metals [4,11]. Then the smooth yield surface is adopted by mainstream research and some advanced non-associated flow rules [2,7,10,12] have been developed to represent corner effects. These rules are constructed to describe the rotation behavior of the plastic strain rate towards the strain rate deviator. However, they have not been widely used in metal forming simulations due to their complexity in expression and implementation [6]. Furthermore, the parameter identification of these rules is also challenging due to the involvement of abrupt strain-path change.

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This paper is structured as follows. Firstly, we review several typical non-associated flow rules representing corner effects, and construct a concise flow rule through mathematical simplification of the general framework. Then a parameter identification method based on the equivalent tangential shear modulus reduction is used to determine the parameters of A6063-T5. Finally, model validation is performed through comparison with the nonlinear tension-torsion experiment.

Non-associated flow rules representing corner effects

A specific type of non-associated flow rules has been proposed to describe corner effects. Assuming the smooth yield surface, the plastic flow rules are constructed to reflect the rotation of the plastic flow direction towards the strain rate direction, and can be expressed as a general framework:

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = \dot{\lambda}\left((1 - \tan\theta^{\mathrm{p}}\cot\theta)\mathbf{n} + \frac{\tan\theta^{\mathrm{p}}}{\sin\theta}\mathbf{l}\right) \tag{1}$$

where $\dot{\lambda}$ is the plastic multiplier. $\mathbf{n}$ is the unit normal of the yield surface, defined by $\mathbf{n} = \mathbf{N}/\|\mathbf{N}\|$ and $\mathbf{N} = \partial f/\partial\boldsymbol{\sigma}$ (f is the yield function). $\mathbf{l}$ is the unit tensor in the direction of the strain rate deviator, which is defined by $\mathbf{l} = \dot{\boldsymbol{\varepsilon}}^{\mathrm{dev}}/\|\dot{\boldsymbol{\varepsilon}}^{\mathrm{dev}}\|$. θ^{p} denotes the angle between the plastic strain rate and the normal $\mathbf{n}$; θ denotes the angle between the strain rate deviator and the normal $\mathbf{n}$, as shown in Fig. 1.

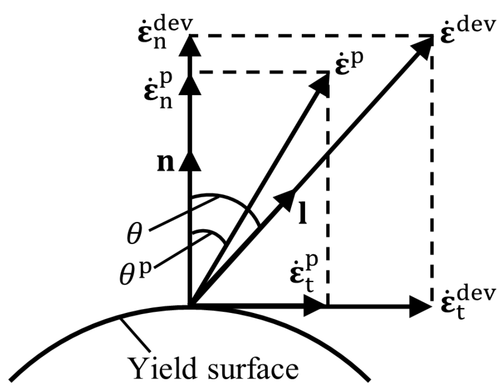

Fig. 1. Schematic of variables and tensors.

Several typical "corner models" can be included in this framework (Eq. 1) as specific forms. For the rule proposed by Simo [12]:

$$\theta^{\mathrm{p}} = \begin{cases} \theta & \text{for } 0 \le \theta < \theta_0 \\ \theta_0 & \text{for } \theta_0 \le \theta \le \pi/2 \end{cases} \tag{2}$$

where θ_0 is a material parameter representing the saturation value of θ^{p}.

The rule proposed by Yoshida [7] approximately satisfies:

$$\tan\theta^{\mathrm{p}} \approx \begin{cases} \tan\theta & \text{for } 0 \le \theta < \theta_0 \\ \frac{\pi/2-\theta}{\pi/2-\theta_0}\tan\theta & \text{for } \theta_0 \le \theta \le \pi/2 \end{cases} \tag{3}$$

where θ_0 is the material parameter.

The rule proposed by Yoshida and Tsuchimoto [2] approximately satisfies:

$$\theta^{\mathrm{p}} \approx \begin{cases} \theta & \text{for } 0 \le \theta < \theta_0 \\ c_3(\theta - \theta_0) + \theta_0 & \text{for } \theta_0 \le \theta \le \pi/2 \end{cases} \tag{4}$$

where c_3 and θ_0 are material parameters.

Here we construct a concise flow rule through simplifying Eq. 1 mathematically. Assuming that $K_\mathrm{c} = \frac{\tan\theta^\mathrm{p}}{\sin\theta}$ is a material constant, the plastic flow rule can be simplified as

$$\dot{\boldsymbol{\varepsilon}}^\mathrm{p} = \dot{\lambda}\left(\left(1 - K_\mathrm{c}(\mathbf{n}:\mathbf{l})\right)\mathbf{n} + K_\mathrm{c}\mathbf{l}\right). \tag{5}$$

Eq. 5 does not contain the variable θ or the branch structure, so the gradient tensor of $\left(1 - K_\mathrm{c}(\mathbf{n}:\mathbf{l})\right)\mathbf{n} + K_\mathrm{c}\mathbf{l}$ with respect to stress can be easily given. Then a stable stress update algorithm is developed for this rule based on the fully implicit backward Euler method, and more information can be seen in the study [13].

Parameter identification

This section presents the parameter identification method of the plastic flow rules representing corner effects. Simultaneously considering the consistency condition, the isotropic Hooke's law and Eq. 1, the stress rate deviator $\dot{\boldsymbol{\sigma}}^\mathrm{dev}$ can be decomposed into two terms, i.e., one depends on $\dot{\boldsymbol{\varepsilon}}_\mathrm{n}^\mathrm{dev}$ and the other depends on $\dot{\boldsymbol{\varepsilon}}_\mathrm{t}^\mathrm{dev}$:

$$\dot{\boldsymbol{\sigma}}^\mathrm{dev} = 2G_\mathrm{n}\dot{\boldsymbol{\varepsilon}}_\mathrm{n}^\mathrm{dev} + 2G_\mathrm{t}\dot{\boldsymbol{\varepsilon}}_\mathrm{t}^\mathrm{dev} \quad \text{with} \quad \begin{cases} G_\mathrm{n} = \left(1 - \frac{1}{\xi}\right)G \\ G_\mathrm{t} = \left(1 - \frac{\tan\theta^\mathrm{p}}{\xi\tan\theta}\right)G \end{cases} \tag{6}$$

where $\xi=1+\frac{H}{2G\|\mathbf{N}\|^2}$ (H is the strain hardening modulus and G is the elastic shear modulus), and its value is close to 1 for common metals since $H \ll G$. G_n and G_t are respectively the coefficients before $\dot{\boldsymbol{\varepsilon}}_\mathrm{n}^\mathrm{dev}$ and $\dot{\boldsymbol{\varepsilon}}_\mathrm{t}^\mathrm{dev}$.

As θ^p is a function of θ, $\tan\theta^\mathrm{p}$ can be expressed as $F(\theta, k_1, k_2, \ldots, k_j)$, where $k_1, k_2, \ldots, k_j$ represent the parameters in a flow rule. Then the equivalent tangential shear modulus G_t can be written as

$$\frac{G_\mathrm{t}}{G} \approx 1 - \frac{F(\theta, k_1, k_2, \ldots, k_j)}{\tan\theta}. \tag{7}$$

The nonlinear tension-torsion loading experiments with abrupt strain-path change are considered for parameter identification. As shown in Fig. 2(a), in Step 1, the tube specimen is stretched until the axial logarithmic strain reaches e_1; in Step 2, the deformation mode is abruptly transformed to combined tension–torsion.

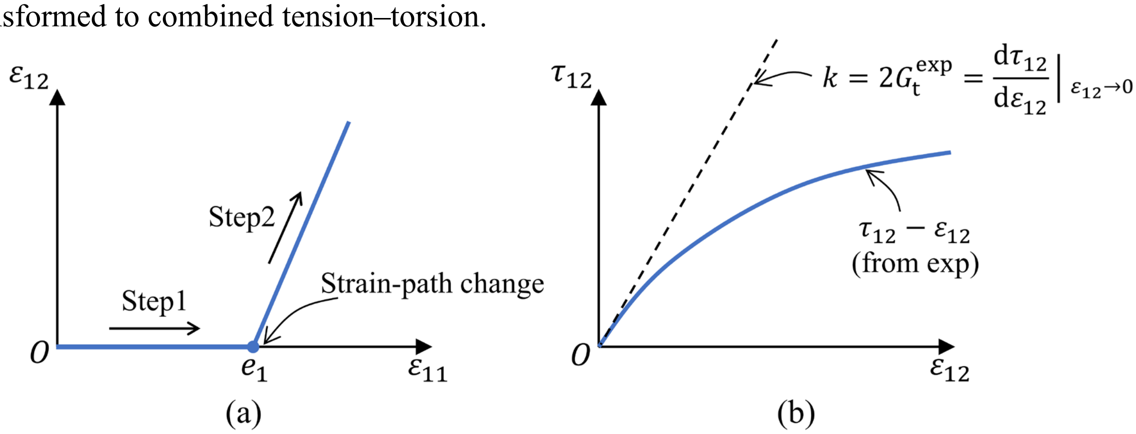

Fig. 2. Schematic of (a) strain path and (b) $\tau_{12} - \varepsilon_{12}$ curve in Step 2.

In the transient immediately after the strain-path change, the ratio of $\dot{\tau}_{12}$ and $\dot{\varepsilon}_{12}$ equals to $2G_t$, as illustrated in Fig. 2(b). Then the values of G_t can be measured and used for parameter identification in Eq. 7. Fig. 3 shows the G_t-data from several nonlinear tension-torsion loading experiments of the aluminum alloy A6063-T5. The parameters of different flow rules are determined through the least square fitting method. The present rule and the rule of Yoshida and Tsuchimoto [2] perform better compared with the associated flow rule and the previous theories proposed by Simo [12], Yoshida [7]. Besides, G_t does not reduce to near zero at least for A6063-T5.

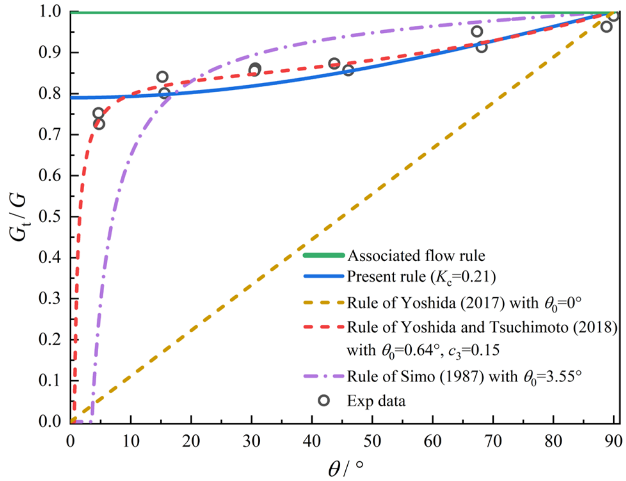

Fig. 3. Experimental data of G_t and predictions of different plastic flow rules.

The above parameter identification method can also be extended to other types of experiments with abrupt strain-path change, and more information can be seen in the study [4].

Model validation

The nonlinear tension-torsion experiment with $\theta \approx 67.5°$ is used for model validation. Here we quantify the plastic flow behavior of A6063-T5 after abrupt strain-path change through the angle β between the plastic strain rate and the loading path trace in the stress space of $\sigma_{12} - \sigma_{11}$, as shown in Fig. 4(a). $\alpha = \arctan\frac{\sigma_{12}}{\sigma_{11}}$ is defined to indicate the position of the current stress point.

The FEM simulations in this section are based on the commercial software ABAQUS. The boundary conditions are applied to a four-node shell element with fully integration. Such unit test simulation is sufficient to completely reproduce the above experimental condition ($\theta \approx 67.5°$). The present rule is implemented by the fully implicit stress update algorithm, while the other flow rules are implemented by the semi-implicit stress update algorithm. And the rule parameters of A6063-T5 have been identified in Fig. 3. The material is assumed to follow anisotropic yielding and isotropic hardening, respectively described by the Poly4 yield function and the Hollomon model.

The results ($\beta - \alpha$ curves) of different plastic flow rules are presented in Fig. 4(b). As the stress path in the early stage of Step 2 is almost the current yield surface, β is predicted to be about 90° by the associated flow rule, and such prediction apparently violates experimental observations. The non-associated flow rules representing corner effects improve the prediction performance of the plastic flow direction after abrupt strain-path change. Among them, the present rule and the rule proposed by Yoshida and Tsuchimoto [2] exhibit the best accuracy.

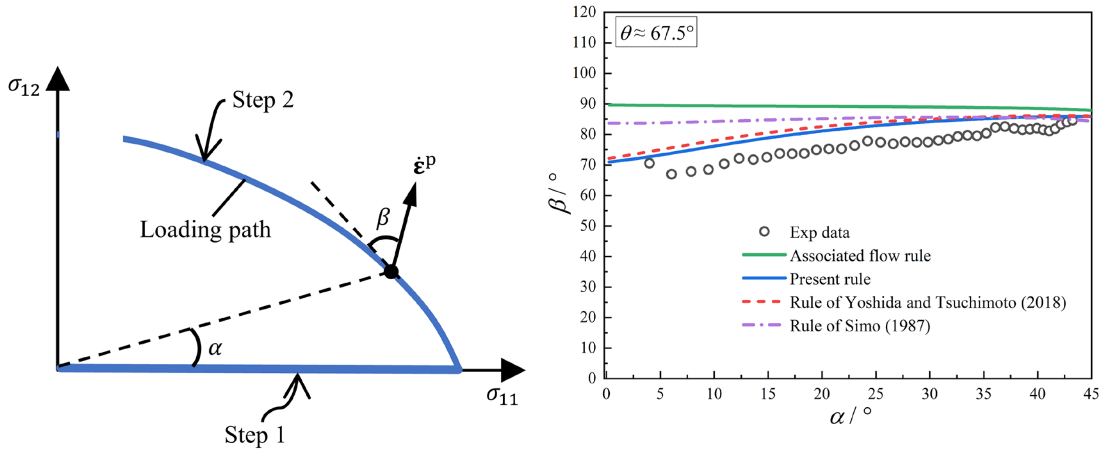

Fig. 4. (a) Schematic of β and α; (b) $\beta-\alpha$ curves predicted by different plastic flow rules.

Conclusions

Several typical non-associated flow rules representing corner effects have been implemented to predict the apparent non-normality of plastic flow in corner effects. The parameters of these plastic flow rules are identified through the reduction of tangential shear modulus. The results suggest that the present rule and the rule proposed by Yoshida and Tsuchimoto [2] exhibit the best accuracy in predicting the flow direction under abrupt strain-path change. Moreover, the tangential shear modulus does not reduce to near zero as reported before.

Acknowledgments

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