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Investigation of radiative needle flow dynamics with variable viscosity and thermal conductivity

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Abstract. The flow pattern formed by the radiative needle moving in a fluid with temperature dependent viscosity and thermal conductivity is investigated in this paper considering the effects of viscous dissipation and nonlinear radiation. Under axial symmetry constraint, the governing equations are converted into a set of non-linear differential equations. Numerical results are presented discussing the influence of Prandtl number, viscosity and thermal conductivity on the heat transfer between the flow and the needle.

Introduction

Boundary layer flows have a pivotal role in aerodynamics affecting phenomena like wing stall, skin friction drag, and heat transfer. In the latter case, which is of scientific and engineering interest for applications as diverse as hypersonic flight, heat exchangers design, or crude oil extraction processes, the boundary layer equations must account for the energy balance equation and the temperature dependence of the viscosity and the thermal conductivity of the fluid. Additionally, at very high temperatures, the magnitude of heat transferred via radiation becomes comparable to that of conduction and convection and should be included as well. Radiative heat transfer also plays a very important role, in general, in flows with high specific enthalpy, in the presence of chemical reactions, dissociation of molecules, ionization of atoms, such as in combustion chambers of chemical rockets [1], in electrical plasma thrusters [2] or during hypersonic atmospheric reentry from extraplanetary space missions or in high enthalpy plasma facilities [3]. However, the effect of radiation on boundary layer flows does not appear fully investigated in the literature, especially when the viscosity and the thermal conductivity are temperature dependent.

The influence of radiation on boundary layer flows over a flat plate was first investigated by Smith [4] upon assuming that convection terms be smaller than conduction terms sufficiently far downstream. Chamkha et al.[5] studied the effect of radiation on the free convection past a vertical plate under the hypothesis of constant viscosity, constant thermal conductivity and linearized heat radiation. Linear radiation effects were also considered by Makinde and Ogulu [6], who however retained the temperature dependence of the viscosity. Pantokratoras [7] investigated linear and nonlinear radiation effects on the natural convection over a vertical plate but considered constant viscosity and thermal conductivity.

The studies above focus on boundary layer flows over flat plates. However, boundary layer solutions have also been found for axial-symmetric problems [8,9]. In the context of radiative heat transfer, Afridi and Qasim [10] recently investigated the effect of nonlinear radiation on the boundary layer flow over a thin needle considering constant viscosity and thermal conductivity.

Based on the literature review above, the present contribution is then intended to study the influence of temperature dependent viscosity and thermal conductivity for a boundary layer flow over a radiative needle. To the best of our knowledge, this problem has not yet been explored.

Problem Analysis
Consider the steady, axisymmetric boundary layer flow over a thin needle assumed as body of revolution, as shown in Fig. 1. The problem is assumed in a cylindrical reference system $O\bar{r}\bar{x}$, whereas $(\bar{r},\bar{x})$ represent the axial and the radial components, respectively. The needle has radius $R = R(\bar{x})$, and is moving with a velocity $\bar{u}_w$ in a free stream of velocity $\bar{u}_\infty$. Under the boundary layer assumptions, the governing equations for this problem are reduced to

$$\frac{\partial(\bar{r}\bar{u})}{\partial\bar{x}} + \frac{\partial(\bar{r}\bar{v})}{\partial\bar{r}} = 0, \tag{1}$$

$$\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{r}} = \frac{1}{\rho}\frac{\bar{\mu}(\bar{T})}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{u}}{\partial\bar{r}}\right) + \frac{1}{\rho}\frac{\partial\bar{T}}{\partial\bar{r}}\frac{\partial\bar{u}}{\partial\bar{r}}, \tag{2}$$

$$\rho\varsigma_p\left(\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{T}}{\partial\bar{r}}\right) = \frac{\bar{\kappa}(\bar{T})}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{T}}{\partial\bar{r}}\right) + \left(\frac{\partial\bar{T}}{\partial\bar{r}}\right)^2\frac{\partial\bar{\kappa}(\bar{T})}{\partial\bar{T}} + \bar{\mu}(\bar{T})\left(\frac{\partial\bar{u}}{\partial\bar{r}}\right)^2 + \frac{16\sigma_{\mathrm{SB}}}{3a_R}\frac{\partial}{\partial\bar{r}}\left(\bar{T}^3\frac{\partial\bar{T}}{\partial\bar{r}}\right), \tag{3}$$

where $\bar{u}$, $\bar{v}$, and $\bar{T}$ are the axial velocity component, the radial velocity component and the temperature of the fluid, respectively.

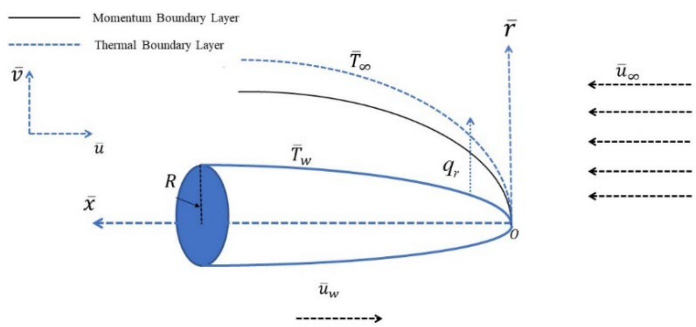

Figure 1. *Flow configuration*

Additionally, ρ is the density, ς_p is the specific heat capacity, $\bar{\mu}$ is the viscosity and $\bar{\kappa}$ is the thermal conductivity of the fluid. As indicated in Eqs. 2 and 3, $\bar{\mu}$ and $\bar{\kappa}$ are functions of temperature; here, the considered functional dependency is

$$\bar{\mu}(\bar{T}) = \frac{\mu_o}{1+\bar{\delta}_\mu(\bar{T}-\bar{T}_\infty)} \text{ and } \bar{\kappa}(\bar{T}) = \kappa_o\left[1 + \bar{\delta}_k\left(\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty}\right)\right], \tag{4}$$

where μ_o, κ_o, $\bar{\delta}_\mu$, and $\bar{\delta}_k$ are material parameters, while $\bar{T}_w$ and $\bar{T}_\infty$ are the needle wall temperature and the free-stream temperature, respectively. The last term in Eq. 3 accounts for the heat exchanged via radiation, where a_R is the Rosseland absorption coefficient and σ_{SB} is the Stefan-Boltzmann constant. The governing equations are closed by the following set of boundary conditions

$$\bar{u} = \bar{u}_w, \bar{v} = 0, \bar{T} = \bar{T}_w \qquad \text{at } \bar{r} = \bar{R}(\bar{x}) \tag{5a}$$

$$\bar{u} \to \bar{u}_\infty, \bar{v} \to 0, \bar{T} \to \bar{T}_\infty \qquad \text{as } \bar{r} \to \infty \tag{5b}$$

Then, considering the following similarity transformations

$$\eta = \frac{\rho(\bar{u}_w+\bar{u}_\infty)\bar{r}^2}{\bar{\mu}\bar{x}}, \bar{\psi} = \frac{\bar{\mu}\bar{x}f(\eta)}{\rho}, \text{ and } \theta(\eta) = \frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty}, \tag{6}$$

it is possible to show that the continuity equation is automatically fulfilled, Eq. 2 reduces to

$$\left(1+\theta\delta_\mu\right)[f''(\eta) + \eta f'''(\eta)] - \eta\delta_\mu\theta' f''(\eta) + \frac{1}{2}\left(1+\theta\delta_\mu\right)^2 f(\eta)f''(\eta) = 0, \tag{7}$$

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while Eq. 3 becomes

$$(1+\delta_k\theta)(\theta'+\eta\theta'')+\eta(\theta')^2\delta_k+\frac{1}{2}\mathrm{Pr}\,\theta' f(\eta)+\frac{4\eta\,\mathrm{Pr}\,\mathrm{Ec}\left(f''(\eta)\right)^2}{\left(1+\theta\delta_\mu\right)}+$$

$$+\frac{4}{3\,\mathrm{N}_r}[\theta(\theta_r-1)+1]^2\left([\theta(\theta_r-1)+1]\left[\eta\theta''+\frac{\theta'}{2}\right]+3\eta(\theta')^2(\theta_r-1)\right)=0 \qquad (8)$$

In Eq. 8, $\mathrm{Pr} = \varsigma_p\bar{\mu}/\bar{\kappa}$ is the Prandtl number, $\mathrm{Ec}=(\bar{u}_w+\bar{u}_\infty)^2/\left(\varsigma_p(\bar{T}_w-\bar{T}_\infty)\right)$ is the Eckert number, $\theta_r=\bar{T}_w/\bar{T}_\infty$ is the heating parameter, and $\mathrm{N}_r=a_R\bar{\kappa}/(4\sigma_{\mathrm{SB}}\bar{T}_\infty^3)$ is the thermal radiation parameter. Eventually, using Eq. 6, the boundary conditions associated with Eqs. 7 and 8 read

$$f(\bar{a})=\frac{\bar{a}\varepsilon}{2}, f'(\bar{a})=\frac{\varepsilon}{2}, \theta(\bar{a})=1 \text{at} \qquad \eta=\bar{a} \qquad (9a)$$

$$f'(\infty)\to\frac{1-\varepsilon}{2}, \theta(\infty)\to 0 \text{as} \qquad \eta\to\infty, \qquad (9b)$$

where $\varepsilon=\bar{u}_w/(\bar{u}_w+\bar{u}_\infty)$ and $\eta=\bar{a}$ identifies the wall surface of the needle.

Results

The set of differential equations given in Eqs. 7 and 8, and their associated boundary conditions given in Eq. 9 are solved numerically. In this study, we use MATLAB built-in bvp5c routine. Some selected numerical results are reported and discussed in this section to investigate the effect of temperature dependent viscosity and thermal conductivity on the heat transfer between the fluid and the radiative needle.

We consider a problem setup identified by $\bar{a}=0.1$, $\varepsilon=0.3$, $\theta_r=2$, and $\mathrm{N}_r=10$, and investigate the effect of the Prandtl number, the Eckert number and the material parameters $\bar{\delta}_k$ and $\bar{\delta}_\mu$ on the heat flux at the needle's wall, i.e. $\theta'(\bar{a})$, and on the local Nusselt number Nu given as

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Re}}}=-2\sqrt{\bar{a}}\left(1+\frac{4}{3\mathrm{N}_r}\theta_r^3\right)\theta'(\bar{a}), \qquad (10)$$

being Re the Reynolds number. The computed results are reported in Table 1, which shows that increasing the Prandtl number leads to an increase of the heat flux; a similar effect is observed if $\bar{\boldsymbol{\delta}}_{\boldsymbol{\mu}}$ increases. On the other hand, increasing $\bar{\boldsymbol{\delta}}_{\boldsymbol{k}}$ or **Ec** lead to a decrease of the heat flux.

Table 1. Effect of the flow parameters on the heat flux between at the needle's wall.

Pr	$\bar{\delta}_k$	$\bar{\delta}_\mu$	Ec	$-\theta'(\bar{a})$	$\mathrm{Nu}/\sqrt{\mathrm{Re}}$
1.2	0.2	0.3	0.5	1.1120	1.4535
3.0	0.2	0.3	0.5	1.4371	1.8784
7.0	0.2	0.3	0.5	1.8358	2.3995
2.0	0.0	0.3	0.5	1.2884	1.6840
2.0	0.3	0.3	0.5	1.2782	1.6707
2.0	0.6	0.3	0.5	1.2678	1.6571
2.0	0.2	0.0	0.5	1.2649	1.6534
2.0	0.2	0.2	0.5	1.2765	1.6685
2.0	0.2	0.4	0.5	1.2864	1.6815
2.0	0.2	0.3	0.0	1.3553	1.7715
2.0	0.2	0.3	0.5	1.2817	1.6752
2.0	0.2	0.3	1.0	1.2082	1.5792

Conclusions

This paper investigates the impact of temperature dependent viscosity and thermal conductivity on the heat transfer between a fluid and a needle considering nonlinear radiation effects in the energy balance equation. The governing equations of the problems are written in axial symmetric coordinates and transformed into a system of nonlinear ordinary differential equations using a similarity transformation. The system of equations is then solved numerically. The obtained results show that removing the hypothesis of constant viscosity or constant thermal conductivity has a non-negligible effect on the heat flux between the fluid and the needle.

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