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Space-based observation modeling method considering resident space object state uncertainty

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Abstract. The proliferation of Resident Space Objects (RSOs) is accelerating at an unprecedented pace, leading to a notable escalation in RSO density and heightened risks to space operations. Consequently, there is an urgent demand for more effective methodologies to estimate the orbital state of RSOs, ensuring the sustainability of space activities. Compared with ground-based observation, space-based observation systems offer significant advantages in accuracy and timeliness owing to their capability for continuous and comprehensive monitoring. However, space-based observation sensors have limited field of view, resulting in their ability to accommodate only a few number of RSOs. Therefore, accurately assessing the detectability of RSOs relative to the sensor and using this information to determine which RSOs to observe is crucial. This can prevent wastage of limited sensor observation resources, thereby acquiring more effective observational data to attain precise RSO states. Due to the inevitable uncertainty in the characterization of RSO states, solely relying on certainty geometric relationships to calculate the detectability is not sufficiently accurate. This paper proposes a space-based observation modeling method that incorporates the uncertainty of the RSO state. By integrating the Monte Carlo (MC) random sampling idea, this method is developed based on probabilistic relationships. This enables the derivation of a numerically continuous distribution of the detectable probability of RSOs, rather than only focusing on whether they are detectable. The intriguing aspect of this research lies in embedding the uncertainty of RSO states into all stages of estimation, thereby enhancing the accuracy of RSO state estimation under limited sensor observation resources.

Introduction

In a space object tracking system, the intrinsic connection between a space object and a moment is the state change of the space object over moments. The space object tracking problem can be modelled as a correspondence between the observation space and the state space of the space object. In the state space, the state of a space object is shifted from the previous moment $k-1$ to the next moment k. However, in the observation space, since only part of the space object state can be observed, a Bayesian recursive method[1] is used to estimate the space object state to obtain a set of space object state information over the moments[2]. Especially with the rapid increase of mega constellations, more RSO state estimates require more efficient us of limited sensor resources[3]. In this process, precisely determining whether a space object can be successfully detected by sensor is essential and challenging.

This research presents a space-based observation modeling approach considering the uncertainty of the space object state. Based on the traditional model relied on geometric relationships, a model reflecting probabilistic relationships is developed through the integration of the Monte Carlo random sampling idea[4]. Compared to traditional space-based observation models, this method embeds the uncertainty of RSO states into various stages of estimation, while

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also generating a continuous numerical distribution representing the detectability probability of space objects. This enhances the robustness of space object state estimation[5].

Space-based observation model based on geometric relationships

Observation sensors typically employ two operational strategies: staring and tracking modes[6]. Both require assessing the detectability of space objects. The process of modelling space-based observations based on geometric relationships is shown in Figure 1.

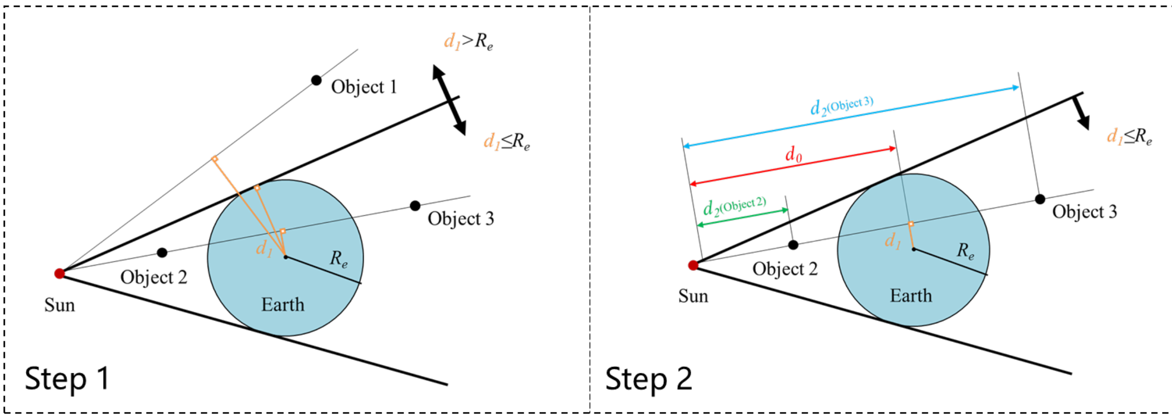

Figure 1 Process for calculating whether a space object is covered by the Earth's shadow.

Whether a space object can be observed by the sensor within the time window is determined by the visibility $p_{v,k}$ of the space object relative to the sensor, which is calculated as

$$p_{v,k} = (1 - p_{s,k})(1 - p_{b,k}) \tag{1}$$

where $p_{s,k}$ indicates whether the space object is covered by the Earth's shadow, obtained from the geometrical relationship between the Sun, the space object and the Earth, and $p_{s,k}$ is 0 when the space object is not in the shadow formed by the Sun on the Earth, and 1 otherwise; and $p_{b,k}$ indicates whether the space object is occluded by the Earth, obtained from the geometrical relationship between the sensor, the space object and the Earth, and $p_{b,k}$ is 0 if the space object is not occluded by the Earth in the field of view of the sensor, and 1 otherwise.

The calculation of $p_{s,k}$ and $p_{b,k}$ is defined as the "0-1 model". The calculation of $p_{s,k}$ consists of two steps. The first step defines the length of the vertical line from the centre of the Earth to the line between the Sun and the space object as d_1, the radius of the Earth as R_e, then $p_{s,k}$ is given by

$$p_{s,k} = \begin{cases} 0, & d_1 > R_e \\ 0 \text{ or } 1, & d_1 \leq R_e \end{cases} \tag{2}$$

When $d_1 > R_e$, the space object is not in the shadow formed by the Sun on the Earth, i.e. $p_{s,k} = 0$. However, when $d_1 \leq R_e$, whether the space object is in the shadow zone will depend on its position relative to the Sun and the Earth. Define the distance from the center of the Sun to line d_1 as d_0, and the distance from the center of the Sun to the space object as d_2. Comparing the lengths of d_2 and d_0 to determine whether the space object is shadowed by the Earth, i.e.

$$p_{s,k} = \begin{cases} 0, & d_2 < d_0 \\ 1, & d_2 > d_0 \end{cases} \tag{3}$$

The calculation of $p_{b,k}$ consists of two steps. The first step defines the length of the vertical line from the centre of the Earth to the line between the sensor and the space object as d_1, the radius of the Earth as R_e, then $p_{b,k}$ is expressed as follows.

$$p_{b,k} = \begin{cases} 0, & d_1 > R_e \\ 0 \text{ or } 1, & d_1 \le R_e \end{cases} \tag{4}$$

When $d_1 > R_e$, the space object is not in a region that may be occluded by the Earth, i.e. $p_{s,k} = 0$. However, when $d_1 \le R_e$, whether the space object is in an Earth-obscured region will depend on its position relative to the sensor and the Earth. Define the distance from the sensor to line d_1 as d_0, and the distance from the sensor to the space object as d_2. Comparing the lengths of d_2 and d_0 to determine whether the space object is occluded by the Earth, i.e.

$$p_{b,k} = \begin{cases} 0, & d_2 < d_0 \\ 1, & d_2 > d_0 \end{cases} \tag{5}$$

Space-based observation model based on probabilistic relationships

Since the large-scale space object tracking problem uses a limited number of space-based optical sensors to track large-scale space objects, it results in a large number of space objects frequently moving in and out of the sensors' field of view. Therefore, correctly calculating the detectability $p_{D,k}$ of a space object to measure the presence or absence of a space object at a given moment k is crucial for accurately estimating the number of space objects. The $p_{D,k}$ is time-varying due to changes in lighting conditions and the relative distance between the sensor and the space object. Therefore, more accurate $p_{D,k}$ prediction model is needed.

The detectability of a space object is determined only by its visibility relative to the sensor. In practical missions, the apparent magnitude of a space object relative to the sensor has a significant impact on its observation quality. The $p_{D,k}$ consists of three components, i.e., the probability of successful detection of a space object as p_D, the apparent magnitude of the space object relative to the sensor as $p_{am,k}$, and the visibility of the space object relative to the sensor as $p_{v,k}$, which is calculated as

$$p_{D,k} = p_D p_{am,k} p_{v,k} \tag{6}$$

where p_D is a constant value that measures the performance of the sensor itself over $[0,1]$; $p_{am,k}$ is calculated from the relative positions of the space object, the sensor and the sun. $p_{v,k} = (1 - p_{s,k})(1 - p_{b,k})$. $p_{s,k}$ and $p_{b,k}$ are determined by the "0-1 model".

Based on the uncertainty of the state of the space object and the probability theory, the "0-1 model" of $p_{s,k}$ and $p_{b,k}$ is refined into the "0~1 model" of $p_{s,k}^{prob}$ and $p_{b,k}^{prob}$ in order to obtain a more accurate $p_{D,k}^{prob}$ with respect to the $p_{D,k}$. Where $p_{s,k}^{prob}$ and $p_{b,k}^{prob}$ are the probabilities of the space object being in the shadow of the Earth and being obscured by the Earth, respectively, and the range of values are $[0,1]$.

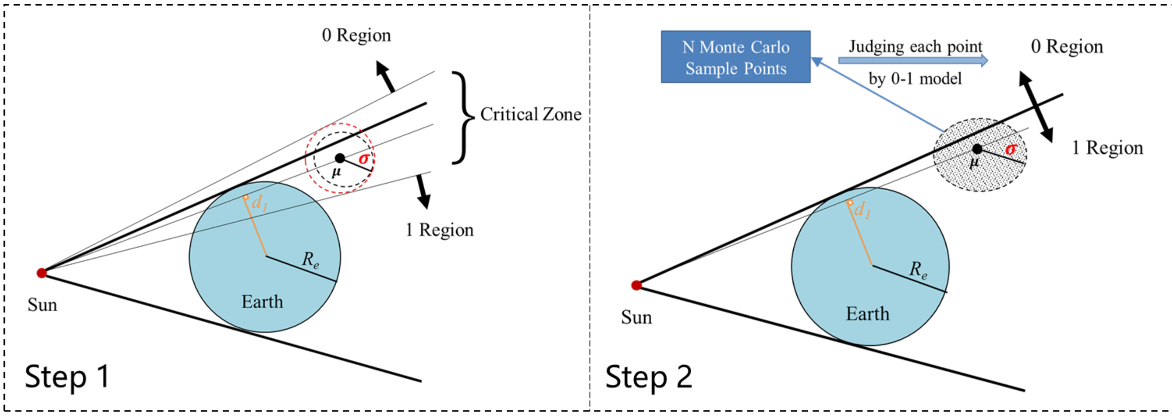

Figure 2 Process for calculating the probability that a space object is covered by the Earth's shadow.

The calculation of $p_{s,k}^{prob}$ requires two steps as shown in Figure 2. The first step defines the maximum two-dimensional standard deviation of the three-dimensional covariance of the space object in the plane of the Sun, the Earth, and the space object as σ. Since the distance between the Earth and the Sun is much larger than the distance between the Earth and the space object, it is approximated whether the space object is in the critical zone by comparing the magnitude relationship between $|d_1 - R_e|$ and $|\sigma|$. Then, the $p_{s,k}^{prob}$ is expressed as

$$p_{s,k}^{prob} = \begin{cases} 0, & |d_1 - R_e| > |\sigma| \text{ and } d_1 > R_e \\ p_{s,k}^{CR}, & |d_1 - R_e| \le |\sigma| \\ 1, & |d_1 - R_e| > |\sigma| \text{ and } d_1 < R_e \end{cases} \tag{7}$$

where $p_{s,k}^{CR}$ is the probability that the space object in the critical zone is in the shadow of the Earth. It can be found that if $|d_1 - R_e| \le \sigma$, then the space object may be in the shadowed or non-shadowed area. Therefore, it is necessary to further calculate $p_{s,k}^{CR}$ for the space objects in critical zone.

Considering the state uncertainty of the space object and the computational complexity, the probability that the space object is in the shadow of the Earth is calculated by combining the Monte Carlo sampling method. For a space object j, N sample points satisfying Gaussian distribution $\mathsf{N}(\mu, \sigma^2)$ are randomly generated in its state distribution space, each sample point represents a space object, and each space object may be located in the shadow area or non-shadow area. From Eq. (2) and Eq. (3), there are N_o sample points located in the shaded area and N_u sample points located in the non-shaded area. Then $p_{s,k}^{CR}$ is calculated by the ratio of N_o and N as

$$p_{s,k}^{CR} = \frac{N_o}{N} \tag{8}$$

Eq. (6) is transformed to

$$p_{D,k}^{prob} = p_D p_{am,k} p_{v,k}^{prob} \tag{9}$$

where the probability that the space object is visible relative to the sensor is calculated as $p_{v,k}^{prob} = (1 - p_{s,k}^{prob})(1 - p_{b,k}^{prob})$.

Up to this point, Eq. (7)-Eq. (9) obtain the space object observation model of the space-based tracking system based on the probabilistic relationship, and obtain the space object detectability probability that is numerically continuously distributed, i.e., realize the prediction of space object

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detectability probability, and improve the accuracy of the state estimation of the space object compared to the existing method of judging the detectability of the space object that is discretely distributed.

Summary

The space-based observation model modelling method considering space object state uncertainty proposed in this paper establishes a space-based observation model based on probabilistic relationships by considering space object state uncertainty based on a space-based observation model based on geometric relationships. The space-based observation model established by this method can obtain numerically continuously distributed space object detectability probabilities, which improves the accuracy and robustness of space object state estimation compared with existing discrete-distributed space object detectability judgement methods.

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