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Multilayer and multifield analysis of origami deployable structures

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Abstract. The research focuses on design of origami deployable structures for space applications. The aim is to acquire new and in-depth knowledge on modeling deployable structures with simple origami folding patterns to enhance the reliability of subsystem design requiring the use of such structures. Mastery of the subject will also enable the analysis of the compatibility of flexible thin structures with electronic components such as flexible rigid PCBs, solar cells, or antennas. The most used method of modelling for the analysis of flexible structures is based on bar-and-hinge models (reduced degree-of-freedom models) which, although accurately describe the macroscopic behavior, are unable to capture local behavior at critical points such as folds and interface points with bonded rigid-flexible elements. Consequently, a refined analysis of the system is carried out by integrating the finite element Carrera Unified Formulation (CUF) in the bar-and-hinge model, which allows to reduce the number of degrees of freedom with respect to classical Finite Element Method (FEM). Such formulation is necessary for two main reasons. Firstly, integrating this formulation with classical reduced degrees of freedom models will allow analyzing the local behavior of the deployable structure while still keeping computational costs low. Secondly, the CUF model is well suited for modeling thin multilayer structures, consisting of layers of very different materials, thanks to the possibility of selecting an arbitrary approximation order along the thickness that is independent of the order of the model adopted in the plane. Such a powerful tool will also allow analyzing the behavior at the interface between distinct layers subjected to both mechanical and coupled thermomechanical stresses, a critical condition in space applications. The final step of the modeling involves research on controlled deployment methods, deepened to ensure greater system reliability and morphing capabilities. The developed models will be experimentally validated through test campaigns aimed at verifying that the stress and strain states resulting from the analyses are comparable to those evaluated in experimental tests. These tests will include, among others, deployment tests of structures with origami patterns or classic folding patterns to evaluate opening stresses and cyclic thermal fatigue tests to evaluate thermomechanical stresses at the interface between distinct layers. The comparison will provide the numerical model with additional robustness and make it a tool capable of predicting complex behaviors otherwise investigable only through experimental tests.

Introduction

The space market has undergone significant transformations in the last two decades. The reduction in satellite launch costs and the pivotal role played by major private players have drastically accelerated the space race in both scientific research and service/business sectors. The future of the space economy appears increasingly promising, thanks to new technologies and growing interest from both agencies and private entities in accessing space.

Research and technological development have accompanied the sector's growth, consistently providing new solutions to the stringent requirements of spacecraft design and reducing production and launch costs.

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In this context, deployable structures and Gossamers play a fundamental role in the design of both small and large satellites. These structures are designed to launch objects with significant volumes and surfaces while remaining compatible with the compactness requirements imposed by launchers. Given the numerous subsystems whose performance depends on their surface extension, these structures find applications in various fields, from power generation using solar panels to signal transmission with antennas, thermal control with thermal shields and radiative panels, and even propulsion and space based solar power generation for ground based business [1].

Although deployable structures are already employed in satellites and spacecraft and are of great technological and market interest, their modelling approach remains a topic of research. Specifically, for extensively large and origami-patterned thin structures, numerical models explaining their behaviour during operational life and deployment are challenging to represent using finite elements, making the computational calculation process cumbersome and complex.
One solution to this issue is provided by reduced order models, which effectively represent the overall behavior of these structures but fall short in capturing local information. In the modeling of origami, the more widely used reduced order model is the bar-and-hinge, consisting of a simplification of the kinematics of foldable thin structure justified by the fact that the big-scale deformations are imposed by the geometry of the folds. The nonlinear elastic formulation for a general bar-and-hinge model introduced by Ke Liu and Glaucio H. Paulino [2] stands as one of the most advanced reduced models.

Despite the accuracy in modeling the global behavior, a finer discretization of the structure is necessary to understand its local behaviors. A finite element shell model seems to be suitable for this purpose, but it presents two significant limitations. Firstly, a shell element models the structure through its thickness as a homogeneous entity, losing information about behavior across the thickness. Secondly, the computational cost of performing nonlinear dynamic analysis of the deployment of rigid-flexible origami structures is significant, especially if multiple shell elements are to be coupled to characterize the behavior of distinct layers across the thickness. There are few studies in the literature that employ finite element models for this purpose, with a clear preference towards reduced element analysis.

Similar issues arise when conducting a multi-field analysis on the structure under consideration, such as thermal-structural coupling induced by thermomechanical stresses, a very common condition in the space field. Uncoupled thermoelasticity models (static, quasi-static, dynamic) are able to model the relationship between stresses/strains and temperature but are not suitable for conditions of high-speed thermomechanical loads. Coupled thermoelasticity models based, such as the one proposed by Green-Lindsay, come into play, allowing to capture the real physical behavior of the component through simulations of the interaction between the mechanical behavior of the elastic body and its temperature where the temporal derivatives of deformations appear in the heat equations. While these equations provide a significant advantage in terms of result accuracy, the coupled thermoelasticity problem entails very high computational costs, and finite element models become necessary. Such models can be further complex in the case of structures with real geometries and very complex boundary conditions, as well as composed of metamaterials, increasingly common in the aerospace field and difficult to model.

Bar-and-Hinge Model

The bar-and-hinge model is based on the principle of stationary potential energy and allows for the development of a nonlinear model in geometry and material characteristics for the analysis of large deformations in origami structures. The panel is discretized into a series of bar and hinge elements typically located at the folds of the pattern or the diagonal of the flexible faces. The discretization just described allows capturing the three fundamental behaviours of origami deformation: stretching, crease folding, and panel bending.

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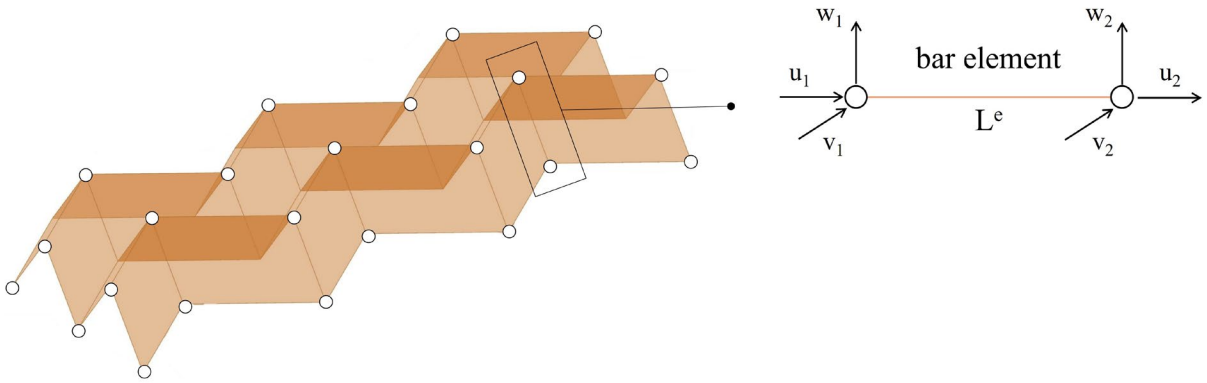

Figure 1: Miura-Ori bar-and-hinge model

In the static case analysis, the potential energy of the system is expressed as:

$$\Pi = U_{bar} + U_{spr} + V_{ext} \tag{1}$$

where U_{bar} is the strain energy of the bar elements, U_{spr} is the energy stored in the springs modelling the folds and the out of plane bending behaviour of the structure, while V_{ext} is the external work. In the dynamic case, the term E_k is introduced in the summation giving the potential energy and represents the kinetic energy of the system:

$$\Pi = U_{bar} + U_{spr} + V_{ext} + E_k \tag{2}$$

In the same way as for the potential energy, the tangent stiffness matrix can be decomposed into two contributions as follows:

$$\boldsymbol{K}(\boldsymbol{u}) = \boldsymbol{K}_{bar}(\boldsymbol{u}) + \boldsymbol{K}_{spr}(\boldsymbol{u}) \tag{3}$$

where $\mathbf{K}_{bar}$ is the stiffness associated with the bar elements while $\mathbf{K}_{spr}$ is the stiffness of the rotational springs modelling even the folding lines or the bending diagonal of the panel.
Each term can in turn be expanded in several matrices:

$$\boldsymbol{K}_{bar}^{(e)} = \boldsymbol{K}_{E}^{(e)} + \boldsymbol{K}_{1}^{(e)} + \boldsymbol{K}_{2}^{(e)} + \boldsymbol{K}_{G}^{(e)} \tag{4}$$

$$\boldsymbol{K}_{spr}^{(r)}(\boldsymbol{u}) = \widetilde{\boldsymbol{K}}_{spr}^{(r)}(\boldsymbol{x}) = k\frac{d\theta}{d\boldsymbol{x}^{(r)}} \otimes k\frac{d\theta}{d\boldsymbol{x}^{(r)}} + M\frac{d^2\theta}{d(\boldsymbol{x}^{(r)})^2} \tag{5}$$

As regards the bars, $\mathbf{K}_E^{(e)}$ is the linear stiffness matrix, $\mathbf{K}_G^{(e)}$ is the geometric stiffness matrix and ($\mathbf{K}_1^{(e)}$ + $\mathbf{K}_2^{(e)}$) form the initial displacement matrix. Concerning spring elements, θ is the dihedral angle of the rotational spring, $\mathbf{x}^{(r)}$ is the vector of nodal coordinates, while k is the tangent rotational stiffness of the element.

Bar and hinge elements can be modelled by different constitutive relations.
Odgen model can be implemented for the analysis of bar elements, according to the following expression:

$$W(E) = \widetilde{W}(\lambda_1, \lambda_2, \lambda_3) = \sum_{j=1}^{N} \frac{\mu_j}{\alpha_j}(\lambda_1^{\alpha_j} + \lambda_2^{\alpha_j} + \lambda_3^{\alpha_j} - 3) \tag{6}$$

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Where λ_i denotes the principal stretches and N, α and μ are the material properties.

On the other hand, rotational hinges for origami structural analysis are usually modelled as linear elastic springs. The momentum generated in response to the external loads is as follows:

$$M = L^{(r)}k(\theta - \theta_0) \tag{7}$$

where k is the rotational stiffness modulus per unit length along the axis and θ_0 is the neutral angle at which the spring is in a stress-free condition. This relation can be generalized to implement the behaviour of nonlinear springs by modelling the stiffness as constant throughout most of its rotation range, while it reaches high values of stiffness in correspondence of a fully packed configuration of the origami.

The resolution of such models relies on numerical computation algorithms such as the Newton-Raphson method. The Merlin model for origami modeling employs a modified generalized displacement control algorithm directly derived from the arc length method.

Carrera Unified Formulation

The solution to the numerical computation problems highlighted in the introductory chapter is provided by the Carrera Unified Formulation (CUF). The choice to integrate the bar-and-hinge model with the finite element CUF is driven by the need to employ a local model for the detailed description of the behavior of deployable structures in space applications without the computational cost hindering analysis in reduced time frames. This formulation allows for deriving the governing equations in a compact way, it can decouple the level of accuracy in modeling along the thickness and in the plane of the thin structure, and it provides accurate solutions with a low number of Degrees Of Freedom (DOFs), without the need to resort to finite 3D elements of higher or lower order.

Below is the formulation of the CUF model for a thin plate. Let's consider a generic plate structure described in a Cartesian coordinate system. Let's consider the mid-plane of the plate lying on the xy-plane, while the thickness of the plate extends along z. The displacement field of a two-dimensional model in the CUF framework is described as a generic expansion of the generalized displacements (in the case of displacement-based theories) by arbitrary functions of the cross-section coordinates:

$$\mathbf{u}(x, y, z) = F_{\boldsymbol{\tau}}(z)\mathbf{u}_{\tau}(x, y) \qquad \tau = 1, \dots, M \tag{8}$$

where $u = \{u_x, u_y, u_z\}$ is the vector of 3D displacements and $u_\tau = \{u_{x\tau}, u_{y\tau}, u_{z\tau}\}$ is the vector of general displacements, M is the number of terms in the expansion, τ denotes a summation and the functions $F_{\boldsymbol{\tau}}(z)$ define the approximation function along the thickness. The generalized displacements are function of the mid-plane coordinates and the expansion is conducted in the thickness direction z.

The same formulation can be applied to thermomechanical and electromechanical problems for the multifield analysis of structures such as piezoelectric ones.

The main advantage of CUF is that it allows to write the governing equations and the related finite element arrays in a compact and unified manner, which is formally an invariant with respect to the $F_{\boldsymbol{\tau}}$ functions

In the case of 2D models, the discretization of generalized displacements on the mid-surface of the plate is made by means of the finite element method or other numerical methods.

Generalized Theory of Thermoelasticity

The CUF model can be employed in the thermomechanical analysis of components subjected to combined structural and thermal stresses [3]. The formulations of coupled thermoelasticity that

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best represent the stress state are those belonging to the category of the Generalized Theory of Thermoelasticity.

According to Green-Lindsay (GL) and Lord-Shulman (LS) theories, the equation of motion of a 3D elastic body in physical coordinates (x, y, z) can be expressed in terms of displacement components as:

$$(\mathrm{C_{ijkl}u_{k,l}})_\mathrm{j} - \left(\beta_{ij}T\right)_j - \left(t_1\beta_{ij}\dot{T}\right)_j + X_i = \rho_\mathrm{i}\ddot{u} + \xi\dot{u}_i \tag{12}$$

Where X_i denotes the body forces per unit volume, ρ is the mass density, ζ is the damping coefficient of the material, u indicates the displacement components vector, C_{ijkl} is a fourth order tensor containing all the elastic coefficients of a general nonhomogeneous anisotropic material, β represents the second order tensor of thermoelastic moduli, T is the temperature change with respect to a reference temperature T_0 and t is time.

On the other hand, the energy equation can be expressed in terms of temperature and displacement fields as:

$$\rho c(t_0 + t_2)\ddot{T} + \rho c\dot{T} - 2\tilde{c}_i\dot{T}_i - \left(k_{ij}T_j\right)_i + t_0T_0\beta_{ij}\ddot{u}_{ij} + T_0\beta_{ij}\dot{u}_{ij} = R + t_0\dot{R} \tag{13}$$

Where k_{ij} is the thermal conductivity tensor, t_0 is the relaxation time associated with LS theory while $\tilde{c}$ is a relaxation time associated with GL theory, c is the specific heat.

The equations just reported constitute the governing system of equations for the generalized coupled thermoelasticity problems.

By applying a finite element formulation through Galerkin approach in a 3D domain, the associated week formulation containing all the possible boundary conditions of the problem is the following:

$$\boldsymbol{\sigma} = \boldsymbol{C\epsilon} - \boldsymbol{\beta}(T + t_1\dot{T}) \tag{14}$$

The computational effort in solving the equation above is quite demanding because of the high number of degrees of freedom. To reduce the computational cost of such a problem without loosing in accuracy, refined 2D models can be implemented in the framework of the CUF presented in the previous paragraph.

In the final system of equation coming from the approximation with CUF approach, the stiffness matrix assumes the following expression:

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_{UU}^{lm\tau s} & \boldsymbol{K}_{U\Theta}^{lm\tau s} \\ 0 & \boldsymbol{K}_{\Theta\Theta}^{lm\tau s} \end{bmatrix} \tag{15}$$

The term K is the thermo-mechanical coupling term that models the mechanical stress induced by a thermal variation. The aim of the initial stages of the research path will be to implement the thermo-mechanical coupling term induced by a mechanical stress.

These coupling terms, combined with the capabilities of the CUF to model multilayer structures, allow for a thorough understanding of the behavior at the interface between two layers of a laminate or the junction between different materials in general [4], thus enabling the prediction of debonding or delamination.

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Preliminary results

The case study considered for analysis is SolarCube, a flexible origami-inspired solar panel designed by Astradyne. The panel lends itself well to the type of analysis described due to the flexible nature of the substrate on which the rigid-flexible electronics are assembled. Moreover, the environment in which it operates is subject to significant temperature gradients, which can induce undesirable phenomena such as excessive relaxation of the substrate or delamination of material layers.

The deployment dynamics will be analyzed using the bar-and-hinge model integrated with CUF and will be compared with results obtained through commercial software such as Ansys and the bar-and-hinge reduced model alone. Figure 2 shows the deployment sequence of the origami with Merlin 2 software (bar-and-hinge).

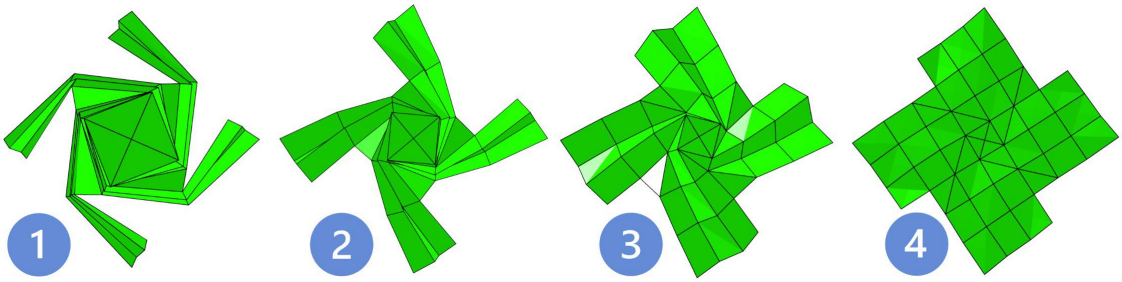

Figure 2: SolarCube's deployment phase. Figure reference: [7]

The structure is discretized with 5 nodes and 8 bars for each face of the origami. As for the material properties assigned, these are partly derived from literature data and partly obtained from mechanical characterization tests conducted in the laboratory. Finally, the deployment dynamics are induced by a displacement constraint at the vertices of the origami. The result is the displacement-force curve in Figure 3, showing a slow and gradual increase in the force required to open the origami and a sudden peak of force corresponding to the deployed configuration.

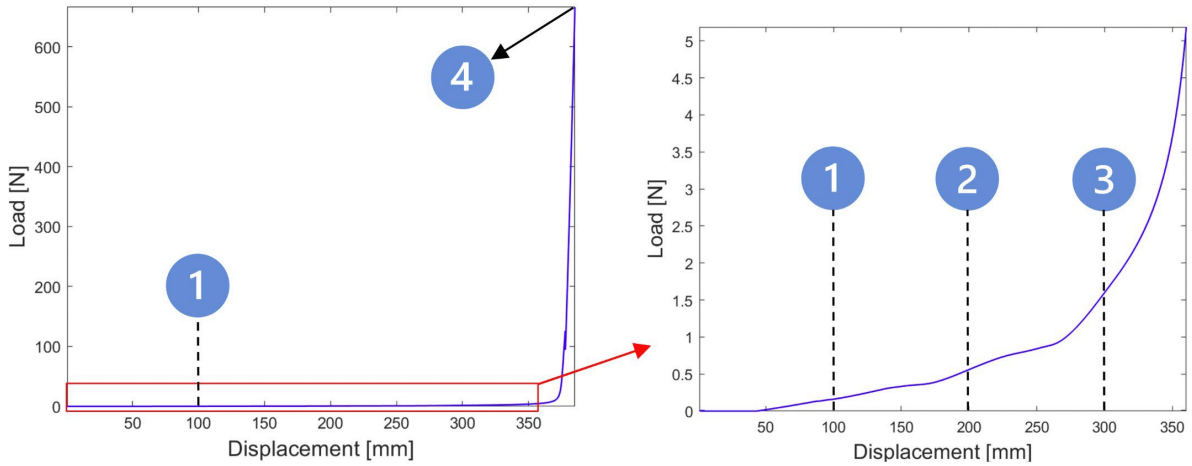

Figure 3: Deployment force path. Figure reference: [7]

The integrated bar-and-hinge and CUF model aims to capture the detail of the thermo-mechanical behavior of the panel at critical local points, potentially subject to delamination or concentrated stress.

Innovation and significant results

The first part of the research will focus on the thermo-mechanical numerical modeling of deployable metasurfaces for space applications. This modeling will include the two thermo-mechanical coupling terms induced by mechanical and thermal stresses, as well as the

nonlinearities of the materials involved, considering the variation of their properties with temperature and fatigue conditions. Such analysis is essential in an environment like space, subject to a large number of thermal cycles over a wide range of temperatures. Further mastery of the results will be acquired by validating the model with experimental tests based on validation processes sanctioned by the ESA (European Space Agency) in the ECSS (European Cooperation for Space Standardization).

Subsequently, finite element models for the dynamics of the deployment of the flexible structure will be carried. The coupling of the CUF with the bar-and-hinge model will allow, unlike common reduced models, to analyze the local behavior of the panel and predict stress concentration points that may cause damage to the panel or delamination between the layers of metamaterial. The CUF and bar-and-hinge model are aimed to preserve the low computational cost of reduced models, while incorporating localized analysis at pre-selected points of interest within the simulation. Moreover, the model is well suited for thermo-electro-mechanical modelling of active deployment systems involving smart material such as shape memory alloys, paving the way for the analysis of innovative deployment systems for origami-inspired deployable space structures. As in the case of the thermo-mechanical model, experimental validation tests of the numerical model will follow to certify the quality of the work done and refine the model to obtain a powerful and effective tool for numerical modeling of structures in both academic and industrial fields.

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