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Fuel-optimal low-thrust CAM with eclipse constraints

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Abstract. This works presents an optimal control direct method to design fuel-optimal low-thrust Collision Avoidance Manoeuvres (CAMs), imposing return to nominal conditions and including eclipse constraints. The methodology exploits the Hermite-Simpon integration scheme to impose the dynamics and the Squared Mahalanobis Distance (SMD) to ensure a collision probability lower than a prescribed value. The choice of using a direct method to solve the Optimal Control Problem (OCP) is justified by the inclusion of orbital perturbations.

Introduction

Satellite Collision Avoidance Manoeuvres (CAMs) play a crucial role in ensuring the safety and longevity of space missions within the increasingly crowded Low Earth Orbit region. As the number of operational satellites continues to rise, there is a growing emphasis on enhancing autonomy in both ground-based and space operations. Managing satellite functions, which are constrained by limited on-board resources, necessitates the deployment of lightweight algorithms that achieve a reasonable balance between efficiency and accuracy. Furthermore, this challenge is rendered more arduous by the increasing use of low-thrust propulsion systems, which necessitate control actions to operate over time intervals rather than relying on instantaneous impulsive manoeuvres.

Designing a CAM generally involves solving an Optimal Control Problem (OCP). The concept of optimality is typically measured in terms of maximising the miss distance, which represents the minimum separation between two approaching objects, or minimising the collision probability. The latter approach is more accurate because it accounts for uncertainties related to orbit determination for the objects of interest.

When dealing with low-thrust propulsion, achieving these optimal solutions often requires continuous, full-throttle thrust. However, energy is a limited resource onboard spacecraft, and minimising waste becomes a primary concern. Excluding natural perturbation exploitation [1], the OCP has to be reformulated as a fuel optimisation problem, ensuring either a minimum miss distance or a maximum probability of collision, also known as Accepted Collision Probability Level (ACPL).

State of the art CAM design

Numerous methods exist in the literature to address low-thrust CAM OCPs. One popular approach is based on the Sims-Flanagan transcription [2], where continuous low-thrust control is approximated as a series of impulsive manoeuvres. The continuous domain is discretised into small arcs such that an impulsive manoeuvre at the centre of the arc would provide the same Δv as if the

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satellite was using low thrust for the whole arc. This discretisation enables the solution of the OCP with Nonlinear Programming (NLP) techniques.

More recently, Bombardelli and Hernando-Ayuso [3] and Gonzalo et al. [4] proposed two similar methods to determine analytically the optimal thrust direction of an impulsive manoeuvre, using Dromo and Keplerian elements, respectively. The main drawback of this formulation is that the model is based on the assumption of zero miss distance at the Close Approach (CA), i.e. the instant at which the separations between the two object is minimum.

A different approach is to solve the optimal control problem using an indirect method. In [5], De Vittori et al. propose an analytical solution of the energy optimal problem through the linearisation of the relative dynamics employing State Transition Matrices (STMs). This solution is then used as initial guess for the fuel optimal problem, which is solved as a Two-Points Boundary Value Problem that targets the desired SMD.

Convex techniques, such as Sequential Convex Programming (SCP), have also been applied to the problem. More recently, the use of AI decision-making tools is becoming increasingly popular. There are many operational aspects that most of the numerically efficient algorithms fail to consider. Particularly, these are the effect of uncertainty on the outcomes from the nominal thrust profile, returning to nominal conditions after the CAM is performed, the effects of orbital perturbations, and thrust constraints during eclipses. The first two problems are tackled separately in [6,7], respectively, while eclipses remain still untreated.

Motivation

The objective of this work is to develop a comprehensive framework for CAMs design, capable of including all relevant features. Notably, these are: fuel optimality, eclipses, perturbations, and return to nominal conditions. The proposed model is intended to serve as a robust foundation for ground-based computations and function as a benchmark for lightweight on-board algorithms.

Model

The proposed model is currently under development and the key design concepts are presented in this section. The dynamics are described in Keplerian elements using Gauss planetary equations, including perturbations, hereby expressed in the form:

$$\dot{\boldsymbol{y}} = f(\boldsymbol{x}, \boldsymbol{a}_c, t)$$

Where $\boldsymbol{x} = (a, e, i, \Omega, \omega, \theta)$ is the vector of Keplerian elements, $\boldsymbol{a}_c$ is the control acceleration, and t is time. Depending on the orbital regime, the perturbing acceleration vector can include the contributions due to geopotential (most notably $J2$, but other spherical harmonics can be considered in resonant regions), atmospheric drag and Solar Radiation Pressure (SRP).

The general OCP is formulated as:

$$\begin{aligned} &minimise && J(\boldsymbol{y}) \\ &subject\ to && h(\boldsymbol{y}) = \boldsymbol{0} \\ & && g(\boldsymbol{y}) \leq \boldsymbol{0} \end{aligned}$$

Where $\boldsymbol{y} = [\boldsymbol{x}, \boldsymbol{a_c}]$ is the optimisation variable composed of state $\boldsymbol{x}$ and control $\boldsymbol{a_c}$, $J(\boldsymbol{y})$ is the objective function, or performance index, to be minimised, and $h(\boldsymbol{y})$ and $g(\boldsymbol{y})$ are, respectively, the equality and inequality constraints. In particular, the aim of the optimisation is to minimise fuel consumption, therefore the performance index is expressed as the integral of the acceleration magnitude over the entire time domain. The initial time t_0 and final time t_f are prescribed as user inputs. In particular, t_0 specifies the first available time to perform the manoeuvre, while t_f is the maximum time allowed to return to nominal conditions.

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The equality constraints are used to impose the dynamics as defect constraints. Particularly, these are imposed exploiting a direct transcription and collocation method, using the Hermite-Simpson integration scheme. The defect constraint δ is the error between the analytical derivative computed at the mid-point of two nodes and the derivative estimated with Hermite-Simpson method at the same point:

$$\delta = \dot{\boldsymbol{x}}_{\boldsymbol{c}} - f(\boldsymbol{x}_{\boldsymbol{c}}, \boldsymbol{a}_{\boldsymbol{c}}, t_c)$$

Where the subscript c refers to the collocation mid-point.

This class of constraint is defined for every pair of adjacent nodes in order to impose the dynamics inside the optimisation procedure.

The second class of equality constraints is used to make the satellite return to nominal conditions after the CAM is performed. This can be achieved by imposing $\boldsymbol{x}(t_f) = \boldsymbol{x}(t_0)$, with the only exception of the true anomaly θ.

Inequality constraints are applied to enforce a minimum SMD at the encounter, and to ensure that the control action is performed outside the eclipse cone.

The SMD is measured on the b-plane and depends on the combined position uncertainty of the approaching objects. It is used in Chan's algorithm [8] to compute the respective collision probability. In particular, if the combined hard-sphere radius is also known, imposing a minimum SMD is equivalent to imposing a maximum ACPL. The respective constraint is defined as:

$$SMD(\boldsymbol{x}, t_{CA}) > \overline{SMD}$$

Where t_{CA} is the time instant of the CA.

To avoid thrusting inside the eclipse cone, a constraint on the control $\boldsymbol{a}_c$ shall also be imposed, where $\boldsymbol{a}_{\boldsymbol{c}} = [a_r\ a_t\ a_n]$ is modelled in the RTN frame (radial, transversal, normal). Re-adapting the methodology presented in [9], it is possible to develop a function $e(\boldsymbol{x})$ that is negative when the satellite is inside the eclipse cone. The eclipse constraint could then be imposed as:

$$-e(\boldsymbol{x})(a_r^2 + a_t^2 + a_n^2) \leq 0$$

Results

At present, progress is confined to the creation of a direct method for evaluating fuel-optimal CAMs. This model, which is based on [1] and [3], uses impulsive manoeuvres to model the low-thrust profile. The optimality of each impulse is denoted by the eigenvalue associated to a specific manoeuvre. The fuel-optimal bang-bang control for low-thrust manoeuvres can be achieved through a one-degree-of-freedom optimisation, where the optimisation variable is the threshold eigenvalue that acts as switching function. Fuel optimal results of impulsive manoeuvres are shown in Fig. 1 for prescribed miss distance, impact parameter and collision probability.

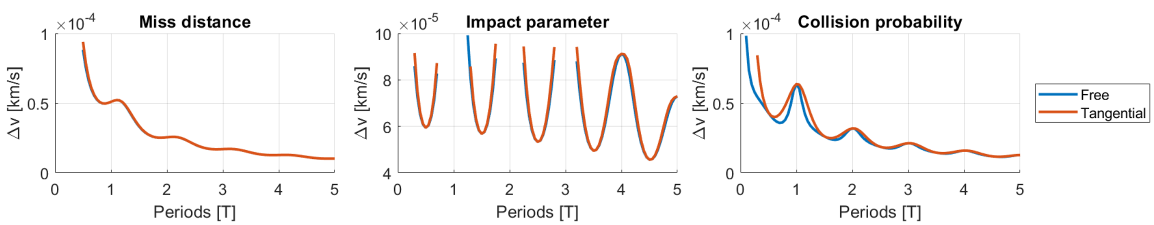

Figure 1 Required $\boldsymbol{\Delta v}$ to obtain a prescribed miss distance of $\mathbf{1}$ km, impact parameter of $\mathbf{250}$ m and collision probability of $\mathbf{10^{-6}}$. The red tangential curves refers to the impulsive being directed along tangential direction only.

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Conclusion
The proposed model presents a novel solution for CAM design, where fuel-optimality, eclipse constraints and return to nominal conditions are included. The methodology exploits direct transcription to express the OPC as an NLP problem imposing both dynamics and operational constraints, in order to provide an efficient and complete decision support module for operators.

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