Materials Research Proceedings 42 (2024) 84-88 https://doi.org/10.21741/9781644903193-19

Coupled thermoelastic analysis using 1D higher-order structural theories and finite elements

Martina Santori[1,a *]

[1]Mul2 Lab, Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

[a]martina.santori@polito.it

Keywords: Thermoelasticity, Carrera Unified Formulation, Finite Element Method

Abstract. This paper presents solutions to coupled thermoelastic dynamic problems of beams subjected to thermal loads over time. A higher-order one-dimensional (1D) model in the framework of the Carrera Unified Formulation (CUF) is used. The study aims to provide accurate predictions for displacement fields and temperature changes within homogeneous isotropic structures under thermal loads. A numerical test case describing the influence of sudden heating on the response of beam structure is presented. The results of the quasi-static analysis are compared with the dynamic response. Different two-dimensional Lagrange expansions are used to discretize the beam cross-section. The approach used in this work simplifies the complex three-dimensional (3D) problem into a computationally efficient 1D model.

Introduction

The study of thermoelastic phenomena has gained significant interest in recent years, given its considerable importance in many engineering applications. In the aerospace sector, thermal stress may be significant given the extreme operating conditions components often face, characterized by high temperatures and rapid temperature fluctuations. The complex interplay between thermal effects and mechanical responses has led to the development of numerous models for studying thermoelastic problems. Static thermal loading involves stationary temperature distributions that affect component deformations, while quasi-static loading accounts for time-dependent temperature changes that lead to transient thermal stresses. On the other hand, dynamic thermoelasticity introduces additional complexities, such as inertial effects, as external thermomechanical loads fluctuate rapidly over time.

Traditionally, thermoelastic problems have been addressed using uncoupled theories, in which temperature and mechanical displacements are treated independently. Although these models offer computational simplicity, they may not accurately predict the behavior of aerospace structures under extreme operating conditions. As such, there is a growing need for more sophisticated theories of coupled thermoelasticity.

In 1956, Biot [1] presented the classical theory of thermoelasticity, according to which thermal disturbances propagate with infinite velocity within the structure. Afterwards, other theories, such as the Lord-Shulman (LS) [2] and Green-Lindsay (GL) [3] models, were developed that overcome the limitations of the classical theory.

The advent of numerical methods, particularly the finite element analysis, has enabled the accurate simulation of complex thermoelastic phenomena. This paper uses a refined 1D model based on the Carrera Unified Formulation [4]. This approach has demonstrated the ability to perform multi-field analysis, particularly thermomechanical analysis, on complex structures, such as beams or disks, in an accurate and computationally efficient way [5,6]. This paper shows how the FE-CUF 1D model allows for accurate coupled dynamic analysis using Newmark's implicit method for resolution. Some numerical results on quasi-static and dynamic problems on an isotropic, homogeneous beam subjected to impulsive thermal loading are presented.

Materials Research Proceedings 42 (2024) 84-88 https://doi.org/10.21741/9781644903193-19

Governing equations

The first general governing equation of the coupled thermoelasticity is the equation of motion in a three-dimensional domain [7]:

$$\sigma_{ij,j} + X_i = \rho\ddot{u}_i + \zeta\dot{u}_i \tag{1}$$

where σ_{ij} is the stress component, X_i is the volume forces and u_i is the displacement component. ρ and ζ are the density and damping coefficient, respectively. The derivative in space is defined by subscript (**,**) while the derivative in time is denoted by superscript ($\cdot$).

The stress component is expressed by Hooke's law for a non-homogeneous anisotropic material as:

$$\sigma_{ij} = C_{ijpq}\epsilon_{pq} - \beta_{ij}(T + t_1\dot{T}) \tag{2}$$

where ϵ_{pq} is the strain component, C_{ijpq} is the 4th-order elasticity tensor, T is the temperature change with respect to the reference temperature T_0 and t_1 is one of the two relaxation times predicted by Green-Lindsay (GL) theory. The parameter β_{ij} è il second-order tensor of termoelastic moduli.

The energy equation can be expressed as a function of displacements u_i and temperature T [7]:

$$\rho c(t_0 + t_2)\ddot{T} + \rho c\dot{T} - 2\tilde{c}_i\dot{T}_{,i} - \left(\kappa_{ij}T_{,j}\right)_{,i} + t_0T_0\beta_{ij}\ddot{u}_{i,j} + T_0\beta_{ij}\dot{u}_{i,j} = R + t_0\dot{R} \tag{3}$$

where c is the specific heat, $\tilde{c}$ is a vector of material constants and t_0 and t_2 are the relaxation times relative to Lord-Shulman (LS) theory and Green-Lindsay (GL) theory, respectively. The parameter κ_{ij} is the thermal conductivity tensor.

Eq. 1 and Eq. 3 represent the coupled governing equations of coupled thermoelasticity written in the most general form. To solve the two equations simultaneously, a finite element formulation is adopted using the virtual displacement principle (PVD) [4]:

$$\delta L_{int} = \delta L_{ext} - \delta L_{ine} \tag{4}$$

where δL_{int}, δL_{ext} and δL_{ine} are the internal, external, and inertial virtual works, respectively.

The displacement field $\boldsymbol{u}$ and temperature variation T can be expressed by CUF using the finite element method through the following relations [4]:

$$\boldsymbol{u} = N_mF_\tau\boldsymbol{U}^{m\tau};\quad T = N_mF_\tau\Theta^{m\tau} \tag{5}$$

where N_m are the shape functions, F_τ are the generic expansion functions and $\boldsymbol{U}^{m\tau}$ and $\Theta^{m\tau}$ are the generalized vector of displacements and the generalized temperature change, respectively. Different Lagrange expansions are used in this work. Using the equations written according to CUF and Hooke and geometric equations within the PVD, the governing equations can be written in the following matrix form:

$$\begin{bmatrix} M_{UU}^{lm\tau s} & 0 \\ M_{\theta U}^{lm\tau s} & M_{\theta\theta}^{lm\tau s} \end{bmatrix}\begin{Bmatrix} \ddot{U}^{ls} \\ \ddot{\Theta}^{ls} \end{Bmatrix} + \begin{bmatrix} G_{UU}^{lm\tau s} & G_{U\theta}^{lm\tau s} \\ G_{\theta U}^{lm\tau s} & G_{\theta\theta}^{lm\tau s} \end{bmatrix}\begin{Bmatrix} \dot{U}^{ls} \\ \dot{\Theta}^{ls} \end{Bmatrix} + \begin{bmatrix} K_{UU}^{lm\tau s} & K_{U\theta}^{lm\tau s} \\ 0 & K_{\theta\theta}^{lm\tau s} \end{bmatrix}\begin{Bmatrix} U^{ls} \\ \Theta^{ls} \end{Bmatrix} = \begin{Bmatrix} F^{ls} \\ Q^{ls} \end{Bmatrix} \tag{6}$$

where the terms of the matrices are expressed through a condensed formulation that does not depend on the order of the model, the so-called fundamental nuclei. For brevity, the expressions for the matrix equation terms are not given in this paper but are explicitly presented in [7].

Newmark's method [8] is used for solving Eq. 6, which in compact form is written:

$$\widehat{\boldsymbol{K}}\boldsymbol{q}_{t+\Delta t} = \widehat{\boldsymbol{R}}_{t+\Delta t} \tag{7}$$

where $\boldsymbol{q}_{t+\Delta t}$ is the vector of the unknowns of displacements and temperature, $t + \Delta t$ is the time step, $\widehat{\boldsymbol{K}}$ is the effective stiffness matrix and $\widehat{\boldsymbol{R}}_{t+\Delta t}$ is the effective loads. The method involves solving the system iteratively for each instant of time. The explicit relations of the terms in the equation and more details of the method can be found in [8].

Numerical results

The case examined is an isotropic beam with a square cross-section clamped at one edge. The beam has a dimensionless length of $\bar{\mathrm{L}} = 0.5$ and a section edge equal to $\bar{\mathrm{l}} = 0.05$. The clamped face is subject to a temperature change $\bar{\mathrm{T}}(\bar{\mathrm{x}}, \bar{\mathrm{y}} = 0, \bar{\mathrm{z}}, \bar{\mathrm{t}}) = 1 - \mathrm{e}^{-100\bar{\mathrm{t}}}$, where $\bar{t}$ is the dimensionless time. The previous parameters are given in the dimensionless form given the very small dimensions and times considered in this case. The relationships between dimensionless and dimensional parameters can be found in [5]. The material is aluminum and has the following characteristics: the Lamè constants are $\lambda = 40.4$ GPa, $\mu = 27$ GPa and $\rho = 2707\ \mathrm{kg\ m^{-3}}$, $\alpha = 23.1 \cdot 10^{-6}\mathrm{K^{-1}}$, $\kappa = 204\ \mathrm{Wm^{-1}K^{-1}}$ [5]. The reference temperature is $T_0 = 293\ K$. The model adopted consists of ten 4-nodes finite elements along the y-axis of the beam and different Lagrange elements to model the cross-section, such as 4-node bilinear (1L4), 9-node biquadratic (1L9), and 16-node bicubic elements.

The quasi-static response of the structure is initially analyzed. The time history of the dimensionless temperature and axial displacement in the midpoint of the structure are shown in Fig. 1.

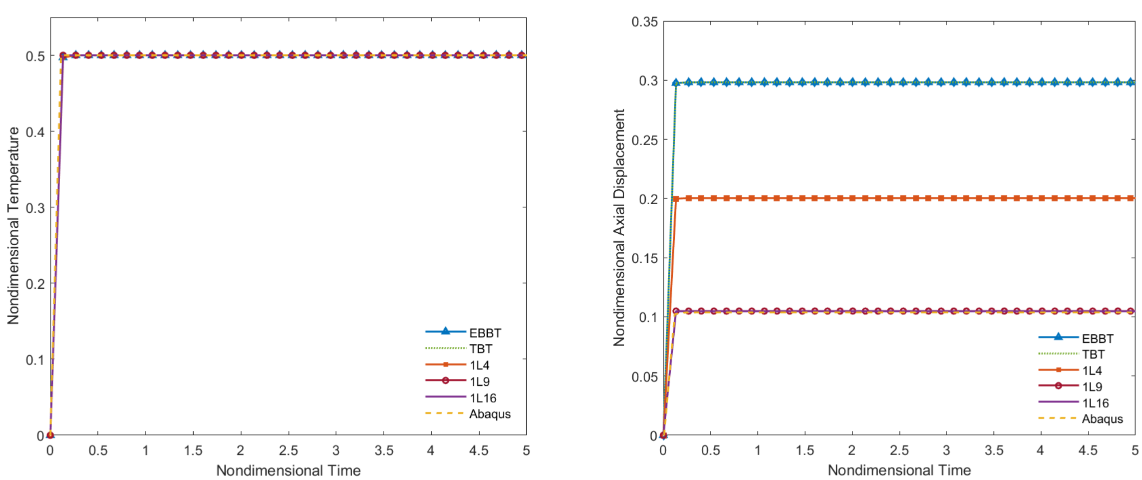

Figure 1: Comparison of nondimensional temperature and axial displacement results of quasi-static analysis obtained through different Lagrange and Abaqus models.

Solutions for different Lagrange models are compared with those obtained from the classical Euler-Bernoulli (EBBT) and Timoshenko (TBT) theories. The results are also compared with the quasi-static response obtained with Abaqus software. The comparison shows that the 1L9 model represents a good trade-off between accuracy and computational efficiency.

In the same time range, the dynamic response using the generalized Lord-Shulman theory is shown in Fig. 2 compared with the quasi-static case. The dimensionless relaxation time of the LS theory is assumed to be $\bar{t} = 0.64$. The inclusion of inertial effects results in fluctuations of temperature and displacement trends around the value obtained from the quasi-static case. In Fig. 2, the dynamic response using the LS model is also compared with the results obtained from the classical dynamic theory. Unlike the classical theory, the LS model manages to predict temperature fluctuations before reaching the steady-state value. Furthermore, the classical theory underestimates the value of the displacement. The trends of temperature and axial displacements are verified through comparison with the solution proposed by Filippi et al. [5] shown in Fig. 2.

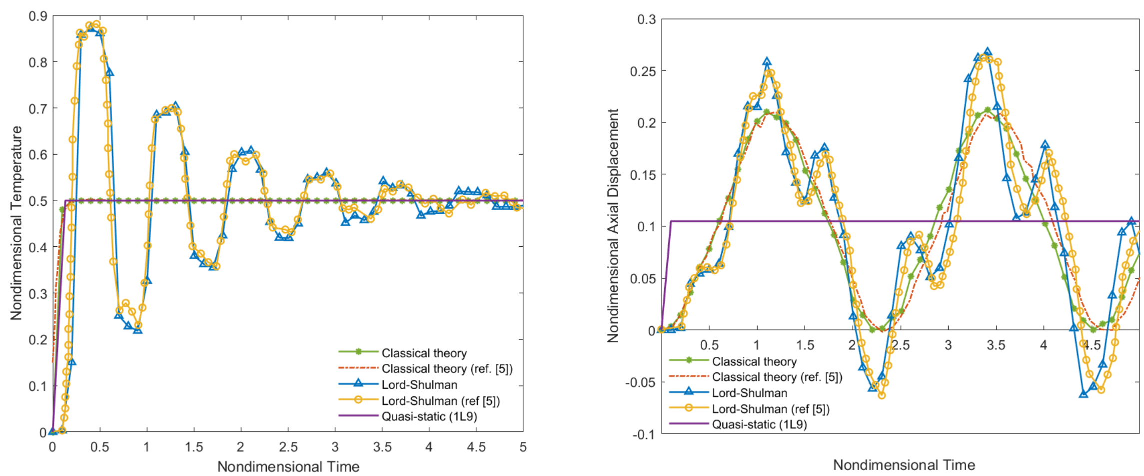

Figure 2: Time history of temperature and axial displacement obtained from dynamic analysis compared with reference results [5] and the quasi-static case.

Summary

This paper presents some numerical results of coupled dynamic thermoelasticity on a homogeneous and isotropic beam. The trends of temperature change and displacements of the structure for the quasi-static and dynamic cases were obtained using a refined 1D model based on the Carrera Unified Formulation (CUF). The approach used was validated through comparison with a commercial code and reference solutions. The results show that by neglecting inertial effects, the quasi-static analysis cannot predict the temperature and displacement fluctuations inferred from the dynamic theory. In addition, using generalized theories such as the Lord-Shulman model allows for a more accurate response of the structure by predicting oscillations in temperature that classical theory could not.

Acknowledgements

This work was partly supported by the Italian Ministry of Foreign Affairs and International Cooperation, grant number US23GR12.

References

[1] M. A. Biot. Thermoelasticity and irreversible thermodynamics. *Journal of applied physics*. 27, 240-253 (1956). https://doi.org/10.1063/1.1722351

[2] H. W. Lord, Y. Shulman. A generalized dynamical theory of thermoelasticity. *Journal of the Mechanics and Physics of Solids*. 15, 299–309, 1967. https://doi.org/10.1016/0022-5096(67)90024-5

[3] A. E. Green, K. A. Lindsay. Thermoelasticity. *Journal of elasticity*. 2, 1–7, (1972). https://doi.org/10.1007/BF00045689

[4] E. Carrera, M. Cinefra, M. Petrolo, E. Zappino. Finite element analysis of structures through unified formulation. *John Wiley & Sons* (2014). https://doi.org/10.1002/9781118536643

[5] M. Filippi, A. Entezari, E. Carrera. Unified finite element approach for generalized coupled thermoelastic analysis of 3D beam-type structures, part 2: Numerical evaluations. *J. Therm. Stresses*. 40.11, 1402-1416 (2017). https://doi.org/10.1080/01495739.2017.1336741

[6] M.A. Kouchakzadeh, A. Entezari, E. Carrera. Exact Solutions for Dynamic and Quasi-Static Thermoelasticity Problems in Rotating Disks. *Aerotec. Missili Spaz*. 95, 3–12 (2016). https://doi.org/10.1007/BF03404709

[7] A. Entezari, M. Filippi, E. Carrera. Unified finite element approach for generalized coupled thermoelastic analysis of 3D beam-type structures, part 1: Equations and formulation. *J. Therm. Stresses*. 40.11,1386-1401 (2017). https://doi.org/10.1080/01495739.2017.1336740

[8] K. J. Bathe. Finite element procedures. Klaus-Jurgen Bathe (2006). https://doi.org/10.1002/9780470050118.ecse159