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Coupling VLM and DG methods for the aeroelastic analysis of composite wings

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Abstract. In this work, a novel computational tool is introduced for rapid aeroelastic analysis of composite wings, integrating an aerodynamic Vortex Lattice Method with a structural Interior Penalty Discontinuous Galerkin method. After the mathematical description of the aeroelastic model, some results for a composite wing are presented to investigate the influence of varying lamination angles on the wing displacement, twist, and divergence speed. Validation against commercial software confirms the effectiveness of the approach.

Introduction

The ability to predict how structures respond to aerodynamic forces is essential for various applications in civil, energy, and aerospace engineering. For instance, modern high-altitude long-endurance aircraft feature very flexible high-aspect ratio wings, prone to large deformations; therefore, their design must carefully account for aeroelastic effects, i.e. the mutual interaction between the deformation of the wing structure and the aerodynamic loads acting on it.

Solving the aeroelastic response of a structure is a fluid-structure interaction problem, which generally cannot be solved analytically and requires computational approaches based on the combined use of numerical codes for structural and fluid mechanics [1, 2]. One solution strategy could involve the coupling of three-dimensional elasticity and fluid-dynamics models, which would offer high-resolution results but involve a high computational cost of the simulations, significantly limiting their use during preliminary design stages. A different approach is to introduce suitable assumptions on the structural model (e.g., representing the wing as a beam component) and on the fluid model (e.g., assuming potential flow), which would reduce the fidelity of the obtained results but also significantly accelerate the numerical simulations.

The approach proposed here falls within the latter category, and its novelty consists in the coupling of an Interior Penalty Discontinuous Galerkin (DG) method for beam structures [3, 4] and the Vortex Lattice Method (VLM) for potential-flow aerodynamics [5].

Aeroelastic Model

Structural model. Consider a composite flat-plate wing of half-span L, chord c, thickness ζ and sweep angle Λ referred to an orthogonal global coordinate system $Ox_1x_2x_3$, with the axis x_1 along the chord and the axis x_3 along the wing thickness, Fig.(1). Small strains and linear elastic stress-strain relationship are assumed, so that

$$\boldsymbol{\gamma} = \boldsymbol{\mathcal{D}}\mathbf{u} \qquad \boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{\gamma} \tag{1}$$

where $\boldsymbol{u} = (u_1, u_2, u_3)^T$ is the vector containing the displacement components, $\boldsymbol{\gamma} = (\gamma_{11}, \gamma_{22}, \gamma_{33}, \gamma_{23}, \gamma_{31}, \gamma_{12})^T$ and $\boldsymbol{\sigma} = (\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{31}, \sigma_{12})^T$ are the vectors containing the strain and stress components in Voigt notation, respectively, $\boldsymbol{\mathcal{D}}$ is the strain-displacement linear differential matrix operator, and $\boldsymbol{C}$ is a 6×6 matrix containing the stiffness coefficients. It is worth noting that the stress-strain relationship is written in the global reference system upon assuming that the material be orthotropic in a local reference system $O\tilde{x}_1\tilde{x}_2\tilde{x}_3$, which is defined such that the axis $\tilde{x}_3$ coincides with the axis x_3 and the axis $\tilde{x}_1$ forms an angle θ with the x_1 axis.

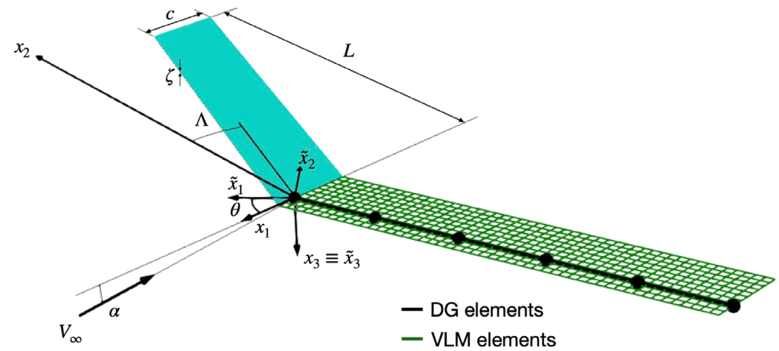


Figure 1. Wing representation.

The wing structure is modelled using high-order generalized beam theories [6, 7, 3] placing the beam elements along the line of quarter-chord points, as sketched in Fig.(1). Following the procedure developed in Ref.[4], it is possible to show the DG weak form of the structural problem reads: find $\boldsymbol{U}^h \in \mathcal{V}^{hp}$ such that

$$B_{\mathrm{DG}}(\boldsymbol{V}, \boldsymbol{U}^h) = L_{\mathrm{DG}}(\boldsymbol{V}, \overline{\boldsymbol{B}}), \qquad \forall \boldsymbol{V} \in \mathcal{V}^{hp}, \tag{2}$$

where $\mathcal{V}^{hp}$ is the space of discontinuous basis functions, $\boldsymbol{U}^h$ is the vector of the generalized displacement components associated with the chosen beam theory and computed using the present DG method, the bilinear form $B_{\mathrm{DG}}(\boldsymbol{V}, \boldsymbol{U}^h)$ accounts for internal elastic energy, and $L_{\mathrm{DG}}(\boldsymbol{V}, \overline{\boldsymbol{B}})$ accounts for the generalized external forces $\overline{\boldsymbol{B}}$ acting on the wing, including the aerodynamic loads. The expression of $B_{\mathrm{DG}}(\boldsymbol{V}, \boldsymbol{U}^h)$ and $L_{\mathrm{DG}}(\boldsymbol{V}, \overline{\boldsymbol{B}})$ can be found in Ref.[4]. It is worth noting that Eq.(2) also includes the kinematic boundary conditions of the wing, which is assumed clamped at its root sections. After numerical integration, computed through standard Gaussian quadrature, the discrete form of Eq.(2) reads as

$$\boldsymbol{K}_{\mathrm{S}}\boldsymbol{X} = \boldsymbol{F} \tag{3}$$

where $\boldsymbol{K}_{\mathrm{S}}$ is the structural stiffness matrix, $\boldsymbol{X}$ collects the coefficients of the DG basis functions, and the vector $\boldsymbol{F}$ collects the associated external forces, which depend on the aerodynamic flow.

Aerodynamic model. The aerodynamic load field is computed using the VLM [5] considering that the lifting surface consists of the geometric surface containing the chord of the wing. Partitioning the lifting surface into a lattice of ring and horseshoe vortices, the VLM framework leads to the following discrete system of equations

$$\boldsymbol{A}\boldsymbol{\Gamma} = \boldsymbol{b} \tag{4}$$

where $\boldsymbol{\Gamma}$ collects the unknown vortex strengths, and $\boldsymbol{A}$ and $\boldsymbol{b}$ are the aerodynamics coefficient matrix and right-hand side, respectively, stemming from flow impenetrability conditions.

Aeroelastic coupling. The aeroelastic coupling is derived upon noticing that, in general, the right-hand side of Eq.(3) depends on the aerodynamic forces and thus on $\boldsymbol{\Gamma}$, while the coefficients matrix $\boldsymbol{A}$ and $\boldsymbol{b}$ depend on the unit normal of the lifting surface and thus on the deformation of the structure. The coupled aeroelastic problem can then written as

$$\begin{cases} \boldsymbol{A}(\boldsymbol{X})\boldsymbol{\Gamma} = \boldsymbol{b}(\boldsymbol{X}) \\ \boldsymbol{K}_{\mathrm{S}}\boldsymbol{X} = \boldsymbol{F}_{\mathrm{A}}(\boldsymbol{\Gamma}) \end{cases} \tag{5}$$

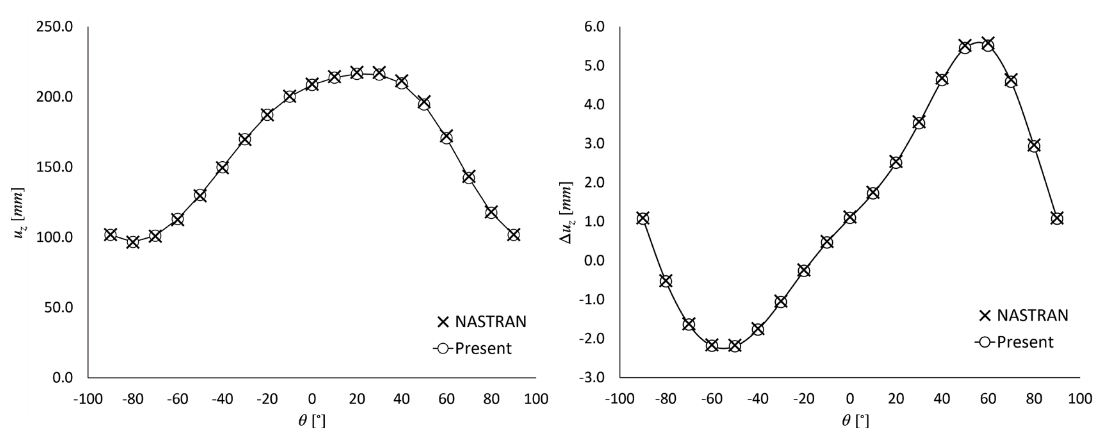
Following the procedure discussed in Ref.[4], the aeroelastic system given in Eq.(6) can be solved by a linearly coupled analysis, which leads to

$$\left[\boldsymbol{K}_{\mathrm{S}} - \rho_\infty V_\infty^2 \left(\frac{\partial \widehat{\boldsymbol{F}}_{\mathrm{A}}}{\partial \boldsymbol{\Gamma}} \boldsymbol{A}^{-1} \frac{\partial \widehat{\boldsymbol{b}}}{\partial \boldsymbol{X}}\right)\Big|_{\boldsymbol{\Gamma}=0,\boldsymbol{X}=0}\right]\boldsymbol{X} = \left(\frac{\partial \boldsymbol{F}_{\mathrm{A}}}{\partial \boldsymbol{\Gamma}} \boldsymbol{A}^{-1}\, \boldsymbol{b}\right)\Big|_{\boldsymbol{\Gamma}=0,\mathbf{X}=0}, \tag{6}$$

where ρ_∞ is the free-stream density, V_∞ is the free-stream velocity magnitude, while $\widehat{\boldsymbol{F}}_{\mathrm{A}}$ and $\widehat{\boldsymbol{b}}$ coincide with $\boldsymbol{F}_{\mathrm{A}}$ and $\boldsymbol{b}$, respectively, computed using $\rho_\infty = 1$ and $V_\infty = 1$. The solution of Eq.(6) provides the aeroelastic response of the structure, whereas solving the eigenvalue problem on the left-hand side of Eq.(6) allows computing the divergence speed.

Results

The proposed framework has been implemented using PySCo[1]. The considered case study is an unswept rectangular flat-plate wing, made of an orthotropic material with half-span $\boldsymbol{L} = \mathbf{10\ m}$, chord $\boldsymbol{c} = \mathbf{1\ m}$ and thickness $\boldsymbol{\zeta} = \mathbf{100\ mm}$. The material properties in the local material reference system are $\boldsymbol{E_1} = \mathbf{20.5\ GPa}$, $\boldsymbol{E_2} = \boldsymbol{E_3} = \mathbf{10\ GPa}$, $\boldsymbol{\nu_{12}} = \boldsymbol{\nu_{13}} = \boldsymbol{\nu_{23}} = \mathbf{0.25}$ and $\boldsymbol{G_{12}} = \boldsymbol{G_{13}} = \boldsymbol{G_{23}} = \mathbf{5\ GPa}$. Fig.(2) shows the effect of the lamination angle $\boldsymbol{\theta}$ on the leading edge vertical displacement $\boldsymbol{u_z}$ and on the twist$\boldsymbol{\Delta u_z}$, respectively, of the tip cross-section using free stream velocity $\boldsymbol{V_\infty} = \mathbf{50\ m/s}$, angle of attack $\boldsymbol{\alpha} = \mathbf{1}°$ and $\boldsymbol{\rho_\infty} = \mathbf{1.225\ kg/m^3}$.



(a) (b)

Figure 2. Effect of the lamination angle θon (a) the leading-edge vertical displacement u_z and (b) on the twist Δu_z of the wing tip cross-section.

The numerical results were obtained using a structural discretization of five eighth-order order DG elements implementing a third-order beam theory, and a VLM lattice composed of $\mathbf{10}\ \times \mathbf{50}$ vortices. The same discretization setup was employed to compute the divergence speed of the wing as a function of the lamination angle θ and three different values of the sweep angle, i.e., $\boldsymbol{\Lambda} = \mathbf{0}°, \mathbf{-10}°, \mathbf{-20}°$, see Fig.(3). The results were compared with those obtained using the commercial software NASTRAN, demonstrating excellent agreement, and validating the proposed model.

[1] https://gitlab.com/aeropa/pysco

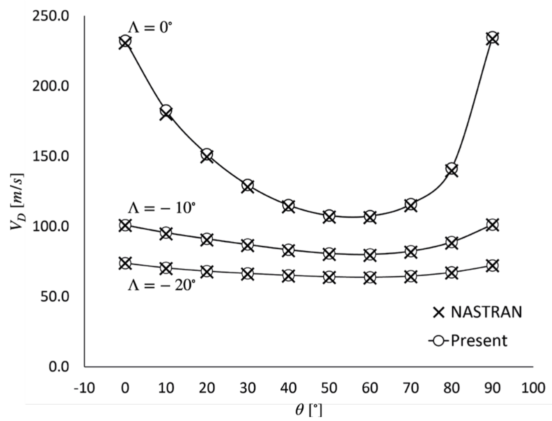


Figure 3. Effect of the lamination angle θ and the sweep angle Λ on the divergence speed V_D.

Conclusions

A computational tool for aeroelastic analysis of composite wings has been proposed. The framework couples an aerodynamic VLM with a structural Interior Penalty DG method for composite wings. The effect of the composite lamination angle on the maximum displacement and twist of the wing tip and on the divergence speed has been studied. The obtained results have been validated by commercial FEM. Further studies will investigate more complex geometries coupled with non-planar VLM approaches and high aspect ratio wings in large-strains.

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