

Nonlinear dynamic modelling and performance of deployable telescopic tubular mast (TTM)

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Abstract. The aim of this work is to present the longitude and transverse vibrations of a deployable TTM, which is attached on a spacecraft system, considering the effect of rigid-flexible coupling phenomenon. The proposed model is derived based on the principle of virtual work and discretized by assumed mode method. To introduce the nonlinear effect, the von Kármán strain is adopted. Additionally, locking and restart behaviors are taken into account in the modelling procedure of the deploying process. Finally, the dynamic phenomena of the longitude and transverse displacements are analyzed at different deploying velocities.

Introduction

As one of the effective ways to achieve large constructions in space, the deployable Telescopic Tubular Mast (TTM) is developed by Northrop Grumman and widely used in the spacecraft systems [1]. The TTM is designed to have several tubes with different radius and each one is nested within one another at the launch stage. When deployed, the tube sections will unfold step by step and locked after each section's motion, experience orbital and attitude rotation at the same time, which results in a hard behavior to model. To solve this problem, a ground test was conducted to determine the deployed stiffness, natural frequencies and its load capability [2]. Moreover, an axially moving cantilever beam model was used when designing the control system to achieve the deployment criteria. However, only the transverse deformation was considered in their work. The longitude motion is also non-neglected when the deployable structures undergoing rotation motions [3]. Therefore, the longitude and transverse vibrations of the TTM during the deployment process are investigated in this work. Moreover, the stepped characteristics of the TTM is considered in this work instead of the uniform beam assumption to make the model more accurate.

Modeling process

The TTM considered in this work is composed of 17 flexible tube sections with different radius and 1 rigid container. All deployable tubes are nested in the container at the initial state and extended sequentially. The spacecraft system contains one rigid central cube hub and two deployable TTMs. Considering the deployable TTM in space as a stepped Euler-Bernoulli beam with axially moving motion (Fig. 1). The length of the deployed structure is $l(t)$, which is varying with time. The other symbols and corresponding values used in this work are listed in Table 1.



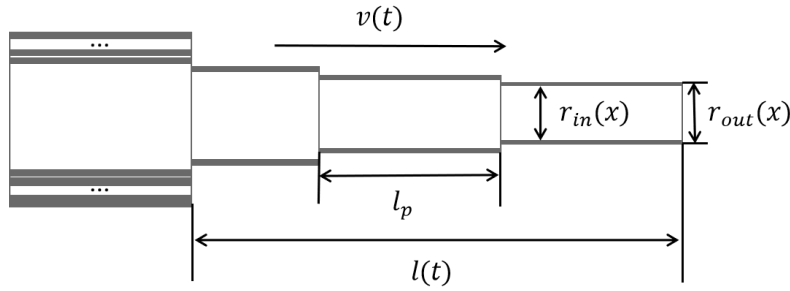


Fig. 1 Deploying process of the TTM

Table 1. Symbols descriptions

Symbols	Description	Value
l_p [m]	length of each section	3
E [GPa]	Young's modulus	70
$I(x)$ [kg·m ²]	moment of inertia	\
ρ [kg/m ³]	density of the boom	2700
$A(x)$ [m ²]	cross section of the boom	\
a [m]	half-length of the central hub	2
r [km]	orbital radius	6700

To describe the nonlinear coupling dynamics of the deployable boom, the von Kármán strain is adopted in this formulation. Therefore, the nonlinear normal strain is expressed as:

$$\varepsilon_{xx} = \frac{\partial u(x,t)}{\partial x} - y \frac{\partial^2 w(x,t)}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w(x,t)}{\partial x} \right)^2 \quad (1)$$

where $u(x,t)$ is the longitude displacement in the x directions, and $w(x,t)$ is the transverse displacement in the y direction. Due to the fact that the two deployable TTMs will not affect each other on flexible vibration [4], only one side is considered in this work. The nonlinear coupling dynamic equations are obtained based on the principle of virtual work:

$$\begin{aligned} & \frac{D^2 u}{Dt^2} + \dot{v} - w\ddot{\alpha} - 2(\dot{\alpha} + \dot{\theta}) \frac{Dw}{Dt} - (a + l_p + x + u)(\dot{\alpha} + \dot{\theta})^2 - \frac{EA}{\rho A} \frac{\partial^2 u}{\partial x^2} \\ & = \frac{\mu}{2r^3} (a + l_p + x)(1 + 3 \cos 2\alpha) \end{aligned} \quad (2)$$

$$\begin{aligned} & \frac{D^2 w}{Dt^2} + (a + l_p + x + u)\ddot{\alpha} + 2\left(v + \frac{Du}{Dt}\right)(\dot{\alpha} + \dot{\theta}) - (\dot{\alpha} + \dot{\theta})^2 w - \left(P_x \frac{\partial w}{\partial x} + P \frac{\partial^2 w}{\partial x^2}\right) \\ & + \frac{EI}{\rho A} \frac{\partial^4 w}{\partial x^4} - \frac{EA}{\rho A} \left(\frac{\partial^2 u}{\partial x^2} \frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2} \frac{\partial u}{\partial x} \right) = -\frac{3\mu}{2r^3} (a + l_p + x) \sin 2\alpha \end{aligned} \quad (3)$$

where α is the attitude angle of the spacecraft system, and θ is the orbital angle. The right sides of Eqs. (2) and (3) represent the perturbation of gravity gradient associated with the rotational motion of the spacecraft system. Using the assumed mode method, the functions for longitude and transverse displacement can be expressed as the summation of mode functions and their corresponding amplitudes:

$$\begin{cases} u(x,t) = \sum_{i=1}^{N_1} \psi_i(x)q_i(t) \\ w(x,t) = \frac{1}{\sqrt{l(t)}} \sum_{j=1}^{N_2} \varphi_j(x)p_j(t) \end{cases} \quad (4)$$

where N_1, N_2 are the truncated order according to the Galerkin's method. The mode shape function can be written as follows according to the boundary conditions [3]:

$$\begin{cases} \psi_i(\varepsilon) = \sin\left(\frac{2i-1}{2} \pi \varepsilon\right) \\ \varphi_j(\varepsilon) = \cosh \lambda_j \varepsilon - \cos \lambda_j \varepsilon - B(\sinh \lambda_j \varepsilon - \sin \lambda_j \varepsilon) \end{cases} \quad (5)$$

where $\varepsilon = \frac{x}{l(t)}$, $B = \frac{\cos \lambda_i + \cosh \lambda_i}{\sin \lambda_i + \sinh \lambda_i}$, λ_i is the i^{th} solution of $1 + \cos \lambda_i \cosh \lambda_i = 0$.

Results and discussions

To analyze the effects of the deploying velocities on longitude and transverse displacements, two constant values denoted as $v = 0.1 \text{ m/s}$ and $v = 0.5 \text{ m/s}$ are adopted in this section when the pause time is 0.05 s , and the spacecraft system remains sun-facing. Here, the pause time represents the time gap after each section is expanded. As a results, the functions of length, velocity and acceleration of the boom are no longer continuous functions due to the existence of time interval. The variations in velocity and acceleration are shown in Fig 2, with the velocity set at 0.5 m/s and time pause time at 2 s the discontinuous characteristics of the deployment process clearly visible.

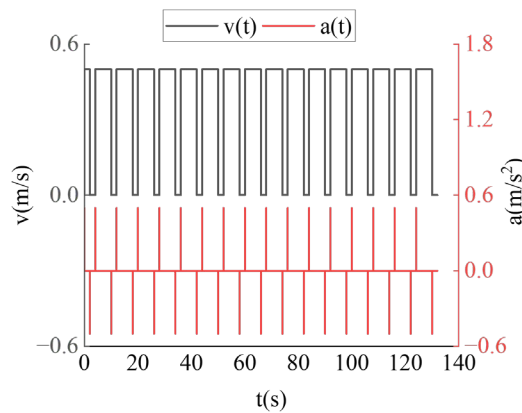


Fig. 2 Velocity and acceleration of the deploying process

The longitudinal and transverse displacements of the tip of the boom under various deploying velocities are depicted in Fig. 3. Throughout the deployment process, the longitudinal displacement remains very small (order of 10^{-3} mm). As the velocity increases, the longitudinal displacement also increases. The intermittent restart and stop actions during the unfolding process of each section lead to sudden changes in amplitude, attributed to the additional pulse acceleration applied along the x -axis. The impact of velocity on transverse vibration is more pronounced, with displacement decreasing as velocity increases. This is because higher velocities correspond to greater transient stiffness of the boom. In addition, the displacement in y -direction is induced by the gravity gradient force. Under the same structural length, the higher deploying velocity induce the lower gravity gradient force since the larger the attitude angle corresponding to the larger sin function when the attitude angle is less than $\pi / 4 \text{ rad}$.

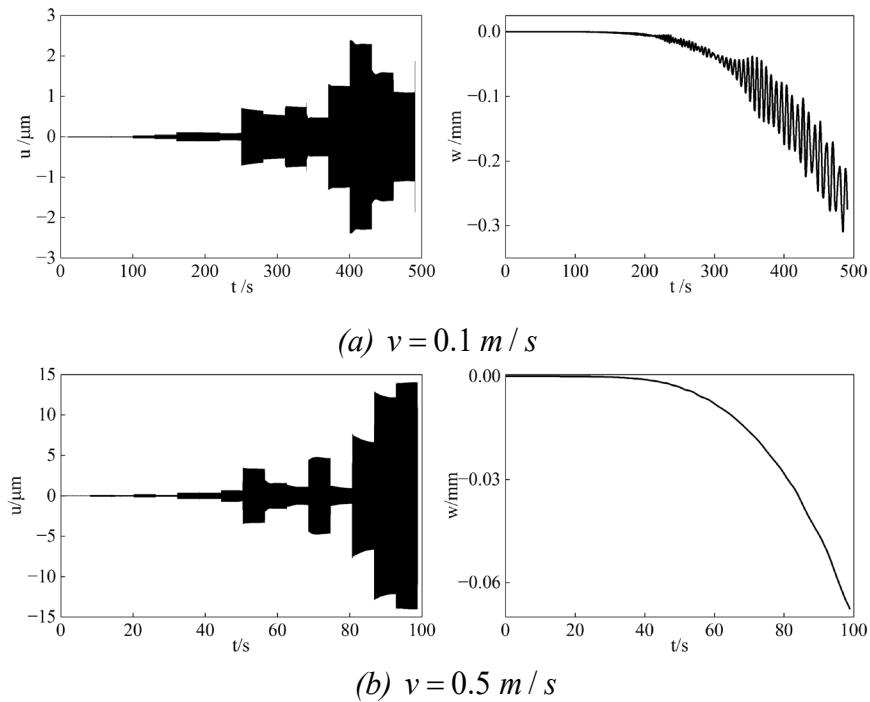


Fig 3. Longitude and transverse displacements during the deploying process for different deploying velocities

Conclusions

The numerical simulation on the unfolding process of TTM is conducted in this work considering the coupling effects of attitude motion. The longitude and transverse displacements are investigated under different deploying velocity. Numerical simulations demonstrate that the longitude displacement increased with the increase of the velocity. However, the transverse displacement exhibit opposite behavior due the properties of dynamical natural frequency and the effects of the gravity gradient force.

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