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Simulation of post-grasping operations in closed-chain configuration using Kane's method

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Abstract. This work focuses on techniques that enable the modeling of a multibody spacecraft in a closed-chain configuration. This scenario pertains to a space manipulator system equipped with two or more robotic arms that have grasped a target object. The dynamic equations of the multibody system, consisting of the chaser and target satellites, are derived using a Kane's formulation for nonholonomic constrained systems. This formulation eliminates the need for including Lagrange's multipliers in the set of equations. Numerical simulations of a post-grasping maneuver between a space manipulator system and a target satellite are conducted to validate the proposed formulation.

Introduction

Modeling multibody spacecraft in closed-chain configurations implies a higher level of complexity with respect to tree configurations. Various approaches are reviewed in Ref. [1], where an extensive comparison is conducted. Among all the strategies presented in this reference, the "cut joint" approach stands out as a compromise between physical consistency, measured in terms of conservation of mechanical energy (in the absence of dissipative effects), and computational efficiency. In this approach, a closed-chain is severed at an underactuated joint to form two separate branches in an open-chain configuration. At the cut joint, a holonomic constraint must be enforced to ensure the compatibility. Different strategies exist to derive the dynamical equations for such systems. When employing a Lagrange's approach, this results in a system of differential-algebraic equations (DAE), where the Lagrange multipliers serve as the adjoint unknowns to recover the internal actions provided by the kinematic constraints. However, a viable alternative is suggested in Ref. [2], introducing a novel version of Kane's equations tailored for constrained systems. This yields a dynamic system divided into two distinct components: an Ordinary Differential Equation (ODE) system comprising motion equations, and an algebraic system of constraint equations. The computation of the latter is only undertaken when explicitly required by the analysis tasks. In the context of this study, this Kane's formulation is employed to simulate a post-grasping scenario. The preparation for repairing a spinning out-of-service satellite is examined, considering a dual-arm space manipulator system (SMS) [3-4]. Initially, the de-spinning process is executed, followed by reducing the relative distance between the target and chaser to a value that permits repairing.

Kane's formulation for constrained systems

The Kane's formulation reported in reference [2] can be applied to model mechanical systems characterized both by kinematic and nonholonomic constraints. However, kinematic constraints must first undergo a time derivation to be transformed into velocity constraints. A multibody system characterized by *n* generalized coordinated q_i (collected in the column vector $\underline{q}$) and *n* generalized velocities u_i (collected in the column vector $\underline{u}$), which are linearly dependent on the time derivative of the generalized coordinates [5], is considered. Then, if *(n-p)* velocity constraints are imposed, the set of n generalized velocities can be divided into two subsets: $\underline{u_I} = [u_1 \quad \cdots \quad u_p]^T$ and $\underline{u_D} = [u_{p+1} \quad \cdots \quad u_n]^T$ containing the independent and the dependent generalized velocities respectively. The constraint equations can be expressed as

$$\underline{u_D} = A\left(\underline{q}, t\right)\underline{u_I} + \underline{b}\left(\underline{q}, t\right), \tag{1}$$

where $A \in \mathbb{R}^{(n-p)\times p}$ and $\underline{b} \in \mathbb{R}^{n-p}$ both depend on the generalized coordinates and time t. It can be proven that, under the constraint equations (1), the Kane's dynamical equations can be written as

$$\underline{\tilde{F}} + \underline{\tilde{F}}^* = A_2\left(\underline{F} + \underline{F}^*\right) = \underline{0} \tag{2}$$

where $\underline{\tilde{F}}, \underline{\tilde{F}}^* \in \mathbb{R}^p$ are the vectors of generalized active forces and generalized inertia forces [2] respectively defined for the constrained system, while $\underline{F}, \underline{F}^* \in \mathbb{R}^n$ are referred to the unconstrained system, and $A_2 = \left[I_{p\times p} \quad A^T\right]$ where $I_{p\times p}$ is the identity matrix of dimension p. The generalized inertia force vector can be rewritten as

$$\underline{F}^* = -M\left(\underline{q}, t\right)\underline{\dot{u}} - \underline{nl}\left(\underline{q}, \underline{u}, t\right) \tag{3}$$

where $M \in \mathbb{R}^{n\times n}$ is the generalized mass matrix, while $\underline{nl} \in \mathbb{R}^n$ is the vector of nonlinear terms of dynamics, i.e. the terms that do not linearly depend on the time derivative of the generalized velocities. Substituting Eq. (3) in Eq. (2) one obtains

$$A_2 M\underline{\dot{u}} = -A_2\underline{nl} + A_2\underline{F}\ . \tag{4}$$

To incorporate constraint equation (1) into the system, it needs to undergo a time derivation to be expressed in the form of acceleration:

$$\underline{\dot{u}_D} = A\underline{\dot{u}_I} + \dot{A}\underline{u_I} + \underline{\dot{b}} \quad \Rightarrow \quad A_1\underline{\dot{u}} = \dot{A}\underline{u_I} + \underline{\dot{b}} \tag{5}$$

where $A_1 = \left[-A \quad I_{(n-p)\times(n-p)}\right]$. Hence, merging of Eqs. (4) and (5) leads to

$$T\underline{\dot{u}} = \left\langle\left[\dot{A}\underline{u_I} + \underline{\dot{b}}\right]^T \quad \left[A_2\left(-\underline{nl} + \underline{F}\right)\right]^T\right\rangle^T \tag{6}$$

where the matrix $T = \left[{A_1}^T \quad \left(A_2 M\right)^T\right]^T \in \mathbb{R}^{n\times n}$ is invertible (the proof is provided in Ref. [2]). Eq. (6) fully describes the motion of a constrained system, yet he does not offer any information about the constraint reactions. However, they can be easily evaluated after solving Eq. (6) through a back-substitution procedure.

Application to the close-chain multibody configuration resolved via cut joint approach

In the framework of Kane's formulation, the dynamical equations for a closed-loop multibody spacecraft can be obtained through the cut joint approach [1]. The process involves opening a closed chain into two open chains at the location of an underactuated joint, while simultaneously enforcing kinematic constraints that are equivalent to the removed joint. The derivation of the dynamical equations is now presented for the specific case of a slider-crank mechanism However, it's worth noting that this approach is applicable to any closed-chain structure.

Materials Research Proceedings 42 (2024) 66-71 | https://doi.org/10.21741/9781644903193-15

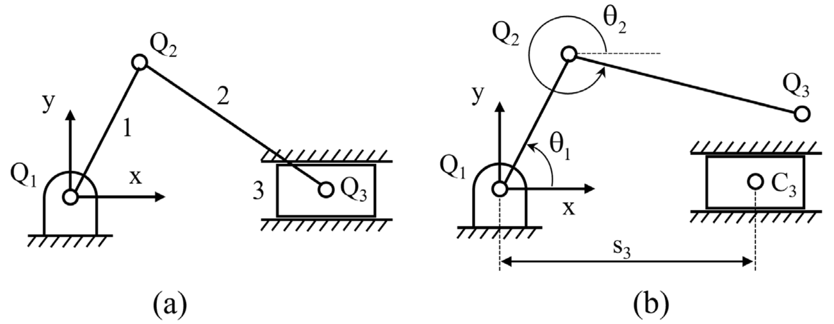

Figure 1: sketch of a slider-crank mechanism (a) before and (b) after the cut open procedure

The sketch of a slider-crank mechanism is depicted in Fig. 1, (a) before and (b) after the cut-open procedure. In the open configuration, the generalized coordinate vector is $\underset{\sim}{q} = [\theta_1 \quad \theta_2 \quad s_3]^T$, where the angles θ_1, θ_2 and the displacement s_3 are depicted in Fig. 1.b, and the generalized velocities are chosen to be the time derivative of the generalized coordinates. Following the Lagrange's or the standard Kane's methodology to derive the system dynamical equations [6], one obtains the same result, i.e.

$$M\dot{\underset{\sim}{u}} + \underline{nl} = \underset{\sim}{F} + \underline{F_{cr}} \tag{7}$$

where $\underline{F_{cr}}$ is the generalized constraint reactions vector associated to the kinematic constraints that must be imposed to guarantee that the position and the velocity of Q_3 always coincide position and velocity of C_3. These constraints can be written in the following form:

$$\underset{\sim}{\Phi}(\underset{\sim}{q}) = \underset{\sim}{0} \quad \Rightarrow \quad \begin{cases} l_1 \cos\theta_1 + l_2 \cos\theta_2 - s_3 = 0 \\ l_1 \sin\theta_1 + l_2 \sin\theta_2 = 0 \end{cases} \tag{8}$$

where l_1 and l_2 are the lengths of the two links. Now, Eq. (8) can be incorporated in Eq. (7) using Lagrange's multipliers or following the Kane's logic described in the previous Section. In the first case, Eq. (8) is derived twice with respect to time to obtain

$$\ddot{\underset{\sim}{\Phi}} \equiv D\dot{\underset{\sim}{u}} = \underset{\sim}{\gamma} \tag{9}$$

where $D \in \mathbb{R}^{(n-p)\times n}$ and $\underset{\sim}{\gamma} \in \mathbb{R}^{n-p}$. For this case study p=1 and n=3. Since the Lagrange's multipliers vector $\underset{\sim}{\lambda} \in \mathbb{R}^{n-p}$ is defined such that

$$\underline{F_{cr}} = -D^T\underset{\sim}{\lambda}\,, \tag{10}$$

one finally obtains the following DAE system:

$$\begin{bmatrix} M & D^T \\ D & 0 \end{bmatrix} \begin{Bmatrix} \dot{\underset{\sim}{u}} \\ \underset{\sim}{\lambda} \end{Bmatrix} + \begin{Bmatrix} \underline{nl} \\ \underset{\sim}{0} \end{Bmatrix} = \begin{Bmatrix} \underset{\sim}{F} \\ \underset{\sim}{\gamma} \end{Bmatrix}, \tag{11}$$

This system is composed of five differential-algebraic equations. On the other hand, when utilizing the presented Kane's method, initially, only one time derivative of Eq. (8) is performed to obtain

$$\dot{\underset{\sim}{\Phi}} \equiv C\underset{\sim}{u} = \underset{\sim}{\beta}, \tag{12}$$

where $C \in \mathbb{R}^{(n-p)\times n}$ and $\underline{\beta} \in \mathbb{R}^{n-p}$. It is worth noting that Eq. (12) can be reconducted to the form of Eq. (1). In fact, for the case of kinematic constraints imposed in the cut joint approach, both $\underline{b}$ and $\underline{\beta}$ are null vectors, so

$$C\underline{u} = [C_I \quad C_D]\begin{bmatrix}\underline{u_I}\\ \underline{u_D}\end{bmatrix} = C_I\underline{u_I} + C_D\underline{u_D} = \underline{0} \quad \Rightarrow \quad A = -{C_D}^{-1}C_I \tag{13}$$

where $C_D \in \mathbb{R}^{(n-p)\times(n-p)}$ and $C_I \in \mathbb{R}^{(n-p)\times p}$. Hence, following the steps described in the previous Section, one obtains a system which expression coincides with Eq. (6). Differently from Eq. (11), the obtained system is composed of three ordinary differential equations that completely describe the motion of the mechanism.

Numerical simulation of post-grasping scenario

In this section, numerical results for a post-grasping scenario are presented. The simulation initiates after the dual-arm Space Manipulator System (SMS) has successfully grasped a target with a mass of 100 kg and a radius of 1 m, which is rotating 5 °/s. Due to the rotational motion of the target, the grasping maneuver was executed after the chaser had synchronized its rotational motion with that of the target. The illustration of the SMS and its physical properties is provided in Fig. 2 and Tables 1-2, respectively. The sequence of operations follows the subsequent steps: first, the chaser performs a de-spinning maneuver while maintaining the target at 1.5 m. This phase lasts for 180 seconds. Then, after 60 additional seconds where the final state of the de-spinning phase is maintained, a manipulation maneuver is carried out. During this maneuver, that lasts 120 seconds, the relative distance between the target and the chaser is reduced from 1.5 m to 0.75 m. The mission concludes after another 240 seconds, during which where the spacecraft maintains the desired final state. Numerical results are reported in Figs. 3-5. The results show that both the de-spinning and the manipulation of the target are successfully achieved within the depicted time span. Furthermore, the control efforts are modest for this mission, facilitating straightforward implementation in terms of actuator sizing.

Concluding remarks

The problem of modeling closed-chain multibody spacecraft has been addressed. The proposed approach relies on (i) a formulation of Kane's equations for constrained systems and (ii) the cut joint procedure. The former enables a reduction in the dimensionality of the equation system while retaining an Ordinary Differential Equation (ODE) structure. Meanwhile, the latter represents a compromise between numerical efficiency and the physical coherence of the results. The process of applying the cut joint technique within the Kane's formulation for constrained systems framework has been outlined. A numerical simulation has been conducted to showcase the capabilities of the presented approach.

Table 1: BUS specifications

Mass [kg]	1000
J_1 [kg m^2]	590.83
J_2 [kg m^2]	270.83
J_3 [kg m^2]	486.67

Table 2: links specifications

Length [m]	1
Radius [cm]	3.5
Mass [kg]	30

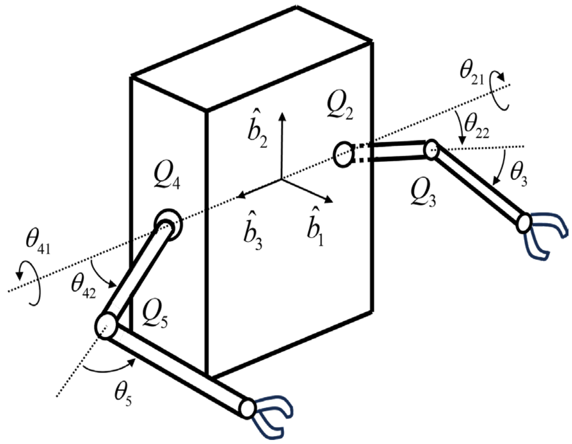

Figure 2: sketch of the Space Manipulator System

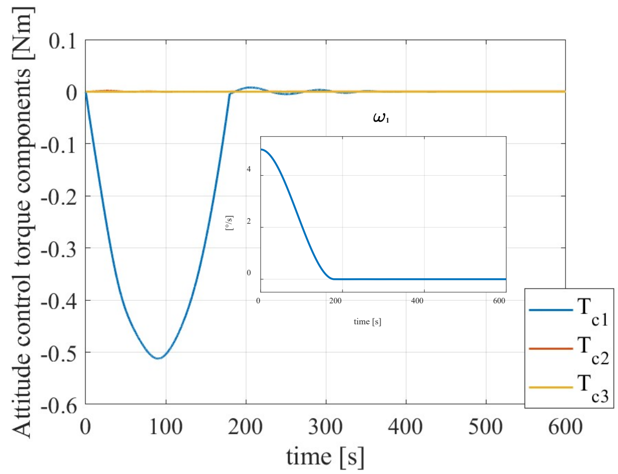

Figure 3: Attitude control torque components and absolute angular rate along b_1

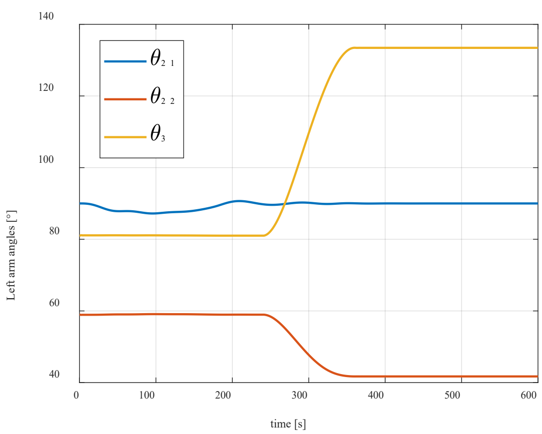

Figure 4: angles of the left arm

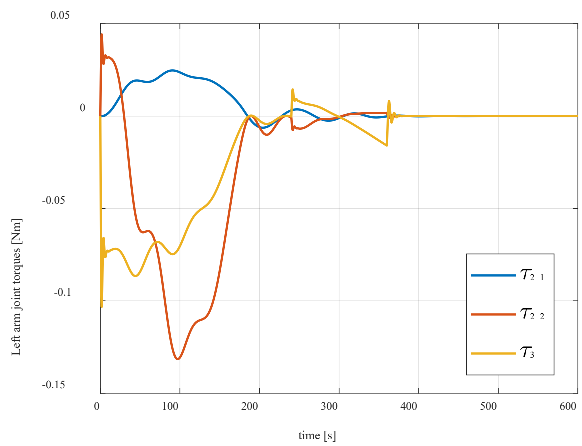

Figure 5: Attitude control torque components

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Materials Research Proceedings 42 (2024) 66-71
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