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Influence of fiber misalignment on the thermal buckling of variable angle tows laminated plates

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Abstract. Variable Angle Tows (VAT) are a class of composite materials with curved fiber paths within the lamina plane. The tailoring of VATs is a virtue that increased the interest in their study. On the other hand, the same freedom in the fiber deposition inevitably leads to manufacturing defects like misalignment that impact the structure behavior, particularly buckling. The present study investigates the influence of manufacturing fiber misalignment on the thermal buckling response of a VAT composite square plate. The governing equations are obtained within the Carrera Unified Formulation (CUF) framework combined with Finite Element Method. The thermal problem is schematized with a decoupled approach, and the critical loads are evaluated through the solution of an eigenvalue problem. The results show how the presence of random misalignment influences both the buckling critical temperature and the buckling mode.

Introduction

The behavior of aerospace structures, particularly those constructed from composite materials, is highly influenced by the thermal environment due to various sources, such as solar radiation or drag in high-speed vehicles. Composites are appreciated in engineering due to their high stiffness and strength-to-weight ratios. They offer tailorable properties and are customizable by acting on stacking sequences or fiber deposition. The introduction of Variable Angle Tows (VAT) further enhances this tailoring capability, enabling curved fiber paths within lamina planes. During these years, the use of VAT has been extensively studied, particularly in the reinforcement around holes [1] and in the optimization of the deposition to retarding mechanical [2] and thermal buckling [3].

While VAT composites present opportunities for improved stiffness and strength properties, their utilization necessitates complex manufacturing techniques, leading to inevitable defects that have a significant impact on some specific phenomena. As a consequence, several studies have been published investigating their influence on failure mechanism [4], mechanical [5], and thermal buckling [6].

This work aims to evaluate the influence of misalignment due to manufacturing on the thermal buckling response of VAT plate, and random fields are employed to consider the random imperfections. The analyses are conducted through a buckling linearized formulation. The thermal problem is schematized using a decoupled approach where the thermal field is assumed to be known at each plate point and treated as an external load. The governing equations are obtained through the Principle of Virtual Displacements (PVD) within the Carrera Unified Formulation (CUF) framework [7] combined with the Finite Element Method (FEM). High-order theories are employed to evaluate the buckling response of the plate, allowing a better description of the results thanks to a more accurate thermal stress evaluation. The buckling results of a thousand analyses are collected, and some statistical conclusions are reported.

Thermal buckling problem

Through the CUF, it is possible to express the principal unknown of the problem which are the 3D displacements $\boldsymbol{u}(x, y, z)$ by the combination of expansion functions $\boldsymbol{F}_{\tau}(z)$ and 2D displacement components $\boldsymbol{u_\tau}(x,y)$. Furthermore, the FEM is employed, and $\boldsymbol{u_\tau}(x,y)$ can be expressed by combining the shape functions $\boldsymbol{N}_i(x,y)$ and the nodal displacement vector $\boldsymbol{q}_{i\tau}$ as represented in Eq. (1).

$$\boldsymbol{u}(x,y,z) = \boldsymbol{F}_\tau(z)\,\boldsymbol{u}_\tau(x,y) = \boldsymbol{F}_\tau(z)\boldsymbol{N}_i(x,y)\,\boldsymbol{q}_{i\tau} \quad \tau = 1,2,\dots,M \quad i = 1,2,\dots,N_n \tag{1}$$

Where the double index means sum, M is the number of expansion terms, and N_n is the number of nodes. More details about the CUF and the allowable expansion functions can be found in [7].

The governing equations are obtained considering a decoupled thermal approach where strain and stresses are decomposed in pure mechanical and thermal components, as reported:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_m + \boldsymbol{\varepsilon}_\theta = \boldsymbol{b}\,\boldsymbol{u} - \boldsymbol{\alpha}\,\Delta T \tag{2}$$

$$\boldsymbol{\sigma} = \boldsymbol{C}\,\boldsymbol{\varepsilon} = \boldsymbol{C}(\,\boldsymbol{b}\,\boldsymbol{u} - \boldsymbol{\alpha}\,\Delta T\,) = \boldsymbol{\sigma}_m - \boldsymbol{\beta}\Delta T \tag{3}$$

Where $\boldsymbol{\beta}$ is a vector containing the terms coupling mechanical and thermal fields, and ΔT is the applied overtemperature. $\boldsymbol{C}$ is the material properties matrix and $\boldsymbol{b}$ is the geometrical operator. More details about these relations can be found in [5].

Through the use of a linearized approach, it is possible to find the thermal buckling critical load as the solution to the eigenvalue problem reported in Eq. (4).

$$\delta\boldsymbol{q}^T :[\,\boldsymbol{K} + \lambda_{cr}\,\boldsymbol{K}_\sigma\,]\,\delta\boldsymbol{q} = 0 \tag{4}$$

Where $\boldsymbol{K}$ is the usual stiffness matrix and $\boldsymbol{K}_\sigma$ is the geometric stiffness matrix that is obtained from the expression of the variation of the work done by the virtual non-linear strains with the thermal initial stresses. The geometric stiffness matrix is assumed to be directly proportional to the buckling critical temperature λ_{cr}.

Misalignment through random field

The fiber path of VAT is described varying linearly according to the notation introduced by Gürdal et al. [8], the lamination is denoted as $[\Phi < T_0, T_1 >]$, where Φ is the reference system rotation angle from x-axis, T_0 and T_1 are the fiber orientation in the center and at the edge of the plate, respectively.

Fiber misalignments are modeled at the layer level through the stochastic field. They belong to a class of defects whose position is uncertain, so a statistical approach is necessary to introduce these defects into the analyses. Misalignment affects the local fiber path, and different orientations must be considered in the corresponding Gauss points.

In the present work, the Karhunen-Loève expansion (KLE) [9] is employed to describe the random fluctuation. Following this expansion, the stochastic field can be expressed as in Eq. (6).

$$\Delta \mathrm{H}^{\mathrm{k}}(x,y,\gamma) = \sum_{i=1}^{\infty} \zeta_i(\gamma)\sqrt{\lambda_i}\;\phi(x,y) \tag{6}$$

Where H denotes the stochastic field and $\Delta\mathrm{H}^{\mathrm{k}}$ is the Gaussian variation of the random field at each layer. ζ_i denotes a set of random variables, λ_i and ϕ are the eigenvalues and the eigenvectors of the autocovariance kernel obtained solving the homogeneous Fredholom integral equation. Among the different solutions of the equation, the exponential function is here employed.

$$C(x,x',y,y') = \sigma_{H^k}^2 e^{-\frac{|x-x'|}{l_x} - \frac{|y-y'|}{l_y}} \tag{7}$$

Where l_x and l_y are the correlation lengths in x and y direction and σ is the standard deviation of the random field.

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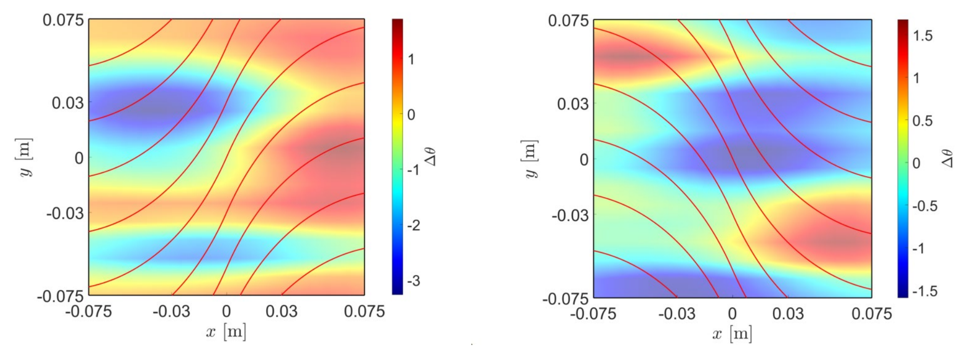
Numerical results

The analysis is focused on a square plate composed of Kevlar/Epoxy with 150 mm edges length and 1.016 mm thickness. The plate is simply supported on all the edges and, one degree overtemperature along the entire plate is applied. Material properties are E_1 = 80 GPa, E_2 = 5.5 GPa, ν=0.34, G_{12} = 2.2 GPa, G_{23} = 1.8 GPa, α_1= -0.9 x 10^{-6} 1/K, α_2= 27.0 x 10^{-6} 1/K. The lamination is $[\pm\theta]_s$ where θ is $[0 < 66.05\,, 11.73 >]$ corresponding to the optimum fiber deposition retarding thermal buckling [3].

Convergence analysis is conducted and 16 x 16 bilinear FEM elements discretization is chosen to ensure a good convergence of the results. Furthermore, a Lagrange Expansion of the second order LE2 is selected as expansion theory along the thickness.

The employed KLE has 15 parameters in the expansion for each layer and, the correlation lengths are $l_x = 0.075$, $l_y=0.0075$. The standard deviation of the random field is σ = 1.5 and the mean of the field is zero. Fig. 1 reports an example of the introduced random misalignment compared to the pristine fiber deposition for the first and second layers of the plate.



(a) **(b)**

Figure 1: Misalignment due to Random Field and pristine fiber deposition. (a) First layer and, (b) Second layer

The first critical load of the pristine plate is equal to 22.02 °C, while the corresponding reference result is 22.28 °C [3], demonstrating the validity of the present method.

In order to investigate the influence of the misalignment on the critical thermal load, a spice of one thousand analyses is selected. The Probability Density Functions (PDFs) for the first five critical temperatures are reported in Fig.s. 2(a) and 2(b), where it is clear that the first and second critical load PDFs are not overlapped and there is a considerable distance between the two critical temperature. The PDFs reported in Fig. 2(b) make clear that the third and fourth buckling loads are overlapped in a certain range of temperatures. To make it clear, Fig. 2(c) reports the number of repetition of the critical load $\boldsymbol{\lambda_3}$ and, $\boldsymbol{\lambda_4}$.

Fig. 3 shows the mean value and the standard deviation of the Modal Assurance Coefficient (MAC) of the buckling modes [5].

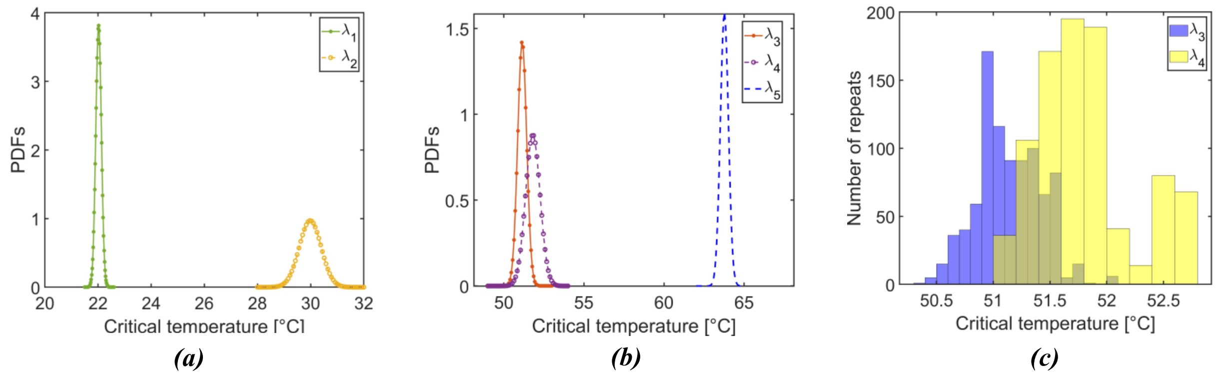


Figure 2: PDFs (a), (b), and number of repetition (c) of the critical temperature for the analyses of the spice.

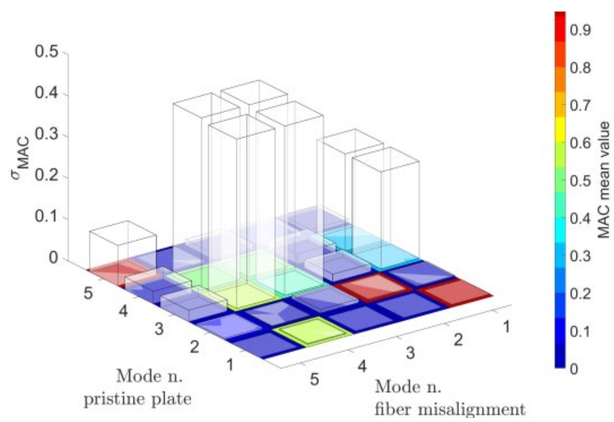


Figure 3: Mean value and standard deviation of the Modal Assurance Coefficient of the analyses.

The standard deviation and mean value of the MAC allow some considerations. The first two modes and the fifth are linearly independent of each other, and there are no interaction phenomena between them. On the other hand, the third and fourth modes present a switch governed by the presence of random misalignments. Therefore, the standard deviation corresponding to these modes is non-zero, and the switching is probably due to the symmetry of load and constraints. Finally, mode 4 presents a combination given by mode 1. This mode consists of a small central deflection with two larger internal deflections, which implies that this mode is not entirely independent of mode 1 that consisting of a single central deflection. The standard deviation of the MAC value is about zero, indicating that this result is present for about each analysis and is independent of the nature of the random misalignment.

Conclusions

The present work focuses on the evaluation of random fiber misalignment on the thermal buckling response of a simply supported square VAT plate composed of Kevlar/epoxy. From the numerical results, it is possible to say that the presence of misalignment does not highly influence the first and second buckling modes of the plate. On the other hand, the third and fourth modes present a switch to the buckling mode and an overlap of the critical temperatures. Future work can be addressed to the study of other materials, plates, or other classes of manufacturing defects.

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