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Stretch forming of isotropic materials: Influence of the ratio between yielding in pure shear and uniaxial tension on the stress state

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Abstract. The formability of metallic sheets could be assessed by performing a hydraulic bulge test. For isotropic materials, interpretation of the bulge test is usually done using the von Mises yield criterion. F.E. simulations of bulge tests were conducted to study how the specificities of the plastic behavior of an isotropic material influences the strain paths and stress paths experienced during hemispherical and elliptical bulging. In this paper, we investigate the role played by the ratio between the yield stresses in pure shear and uniaxial tension of the material, τ_y/σ_T on the mechanical behavior during hydraulic bulging. To this end, we make used of the Drucker yield criterion, which can describe the mechanical behavior of isotropic materials with different ratios τ_y/σ_T with a unique parameter c. For $c = 0$, the Drucker criterion reduces to the von Mises yield criterion (i.e. $\tau_y/\sigma_T = 1/\sqrt{3}$) while for $c \neq 0$, it involves dependence on the third invariant of the stress deviator, J_3. Finite element predictions using the yield criterion reveals a correlation between the ratio τ_y/σ_T which is uniquely defined in terms of the parameter c and the level of plastic strains that develop in the dome, the thickness reduction at the top of the apex, and the strain paths achieved in an elliptical bulge test.

Introduction

Several experiments could be used to extract stress-strain relationship of the metallic materials, the most common one being the tensile test. However, in a tensile test, the range of strains that can be achieved prior to the onset of necking could be limited. A major advantage of the bulge test is that the nature of deformation is such that in most materials, non-uniform strains in the gage area, occurs for larger deformation compared to the tensile test. Therefore, the hardening characteristics of the material may be established over a larger range of effective strain than in the tensile test [1].

The hydraulic bulge test, with either hemispheric or elliptic dies, is also a common method to evaluate the formability of metallic sheets experimentally, due to relatively simplicity of the testing setup (e.g. see [2]–[4]). In a bulge test, a sheet sample is held between two dies and subjected to a hydraulic pressure, which makes the material bulge out of the die opening and form a dome. Since radial deformation is prevented, the sample thickness decreases until it bursts and fails.

A great deal of efforts has been devoted to the theoretical and numerical analysis of the plastic strains and instabilities that develop under hydrostatic bulging. Generally, the test results are interpreted assuming isotropic behavior and yielding governed by the von Mises yield criterion [5]. However, the assumption of plastic behavior governed by the von Mises yield criterion [5] may lead to an overestimation of how much a specimen can bulge before it fails. For isotropic materials that display tension-compression asymmetry on yielding, a new solution to the problem and a correction to the classical relation generally used to determine the stress-strain curve have been derived by Cazacu and Revil-Baudard [6]. However, for textured sheets, the stress state at the top of the bulge can only be determined with the use of a constitutive model for the plastic behavior of the material. Furthermore, for orthotropic metallic sheets, the assumptions of an equibiaxial stress state at the top of a hemispherical bulge could lead to significant error in the analysis of the test.

In this paper, an analysis of the bulge test for isotropic materials with the same yielding response in tension and compression, but different ratios between the yield stresses in pure shear and uniaxial tension of the material, τ_y/σ_T is provided. F.E. simulations of bulge tests were conducted to study how the specificities of the plastic behavior of an isotropic material influences the strain paths and stress paths experienced during hemispherical and elliptical bulging. Specifically, the yielding of the isotropic material is described by the Drucker [7] yield criterion that involve a dependence of the yielding on the third invariant of the stress deviator, J_3. This yield criterion involves a unique parameter c, which is solely expressible in terms of the ratio between the yield stress in simple tension and that in shear τ_y/σ_T. Only in the case when $c = 0$ (i.e. $\tau_y/\sigma_T = 1/\sqrt{3}$), the Drucker yield criterion reduces to the von Mises criterion while for $c > 0$, the material yielding is characterized by $\tau_y/\sigma_T < 1/\sqrt{3}$, while for $c < 0$, the material is characterized by a ratio in yielding between pure shear and uniaxial tension $\tau_y/\sigma_T > 1/\sqrt{3}$. Simulations of bulging tests of various die geometries were conducted for isotropic materials characterized by various values of the ratio τ_y/σ_T. The paper is organized as follows. In section 2, we present the constitutive model, while the F.E. predictions of bulge tests for a variety of materials to provide the evolution of plastic strains and their distribution during the test and better understanding of the state of stress and deformation developing during elliptical bulge tests, are given in section 3. A summary and concluding remarks are presented in section 4. As concerns the notations and conventions used, tensile stresses and strains are positive; the double-contracted product between any two second–order tensors is defined as $\mathbf{M}:\mathbf{N}=M_{ij}N_{ij}$, i, j = 1…3.

Constitutive modeling

Our goal is to study how the yielding description affects the data analysis and the overall behavior of isotropic materials under biaxial stretching deformation. For this purpose, we use an elastic-plastic model with yielding based on Drucker's [7] yield criterion in conjunction with an associated flow rule, and isotropic hardening law. The total rate of deformation $\mathbf{D}$ is the sum of the elastic part $\mathbf{D}^e$ and the plastic part $\mathbf{D}^p$. Linear elastic and isotropic behavior governed by the Hooke's law is used to describe the elastic behavior. The stress-strain relation is given by

$$\boldsymbol{\sigma} = \boldsymbol{C}^e(\boldsymbol{D} - \boldsymbol{D}^p) \tag{1}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor. In Eq.(1), C^e is the fourth-order stiffness tensor, which, for an isotropic material, is expressed with respect to any coordinate system as

$$C^e_{ijkl} = 2G\delta_{ik}\delta_{jl} - (K - 2G/3)\delta_{ij}\delta_{kl} \tag{2}$$

where *i, j, k, l =1…3*, δ_{ij} is the Kronecker unit delta tensor, and *G* and *K* are the shear and bulk moduli, respectively. The plastic part of the strain tensor is determined using the associated flow rule:

$$\boldsymbol{D}^p = \dot{\lambda}\frac{\partial \mathrm{F}}{\partial \boldsymbol{\sigma}} \tag{3}$$

where F is the yield function and $\dot{\lambda}$ is the plastic multiplier. Therefore, the plastic potential F in Eq. (3) is of the general form:

$$\mathrm{F}(\sigma, \bar{\epsilon}^p) = \bar{\sigma}(\sigma) - Y(\bar{\epsilon}^p) \tag{4}$$

where $\overline{\sigma}$ is the effective stress associated with the Drucker [7] yield criterion, $\overline{\varepsilon}^{p}$ the corresponding work-equivalent accumulated plastic strain and $Y(\overline{\varepsilon}^{p})$ is the isotropic hardening law. The effective stress associated with the Drucker [7] yield criterion is given by

$$\overline{\sigma} = B[(J_2^o)^3 - c(J_3^o)^2]^{1/6} \tag{5}$$

In the above equation, $J_2 = 1/2\, tra(\sigma^2)$ and $J_3 = 1/3\, tra(\sigma^3)$ are the second and third invariant of the stress deviator, respectively. B is a constant defined such that the effective stress reduces to the yield stress for uniaxial tension

$$B = \frac{3}{(1 - 4c/27)^{1/6}} \tag{6}$$

In Eq. (5) and Eq. (6), c is a parameter that is solely expressible in terms of the yield stress in the uniaxial tension, σ_T and the yield stress in pure shear, τ_Y, as

$$c = \frac{27}{4}\left[1 - \left(\frac{\tau_{Y\sqrt{3}}}{\sigma_T}\right)^6\right] \tag{7}$$

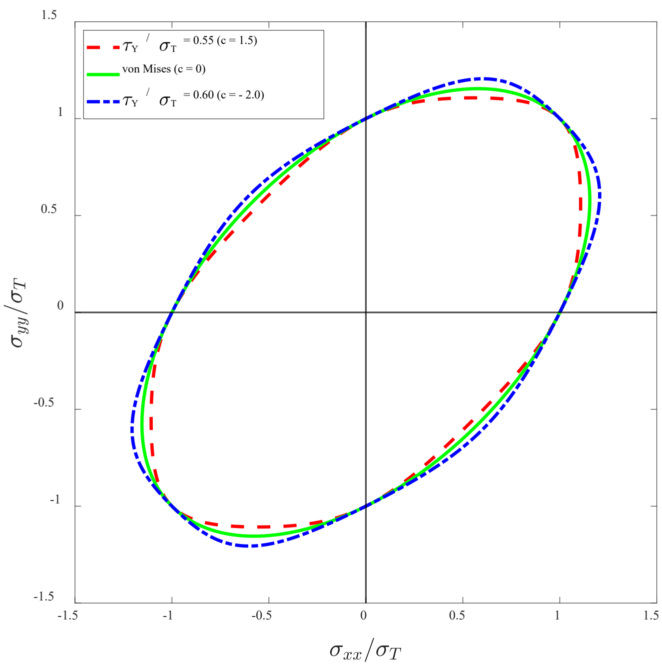


Fig. 1. Projection in the $(\sigma_{xx}, \sigma_{yy})$ plane of the yield loci according to the isotropic Drucker [7] criterion corresponding to $\tau_y/\sigma_T = 0.55$ ($c = 1.5$), $\tau_y/\sigma_T = 1/\sqrt{3}$ ($c = 0$, von Mises material) and $\tau_y/\sigma_T = 0.60$ ($c = -2$), respectively. Stresses are normalized by the yield stress in uniaxial tension, σ_T

The convexity of the yield surface for the Drucker yield criterion is ensured for $27/8 \leq c \leq 2.25$. Hence, the Drucker [7] yield criterion can be used to describe the yielding behavior of isotropic material for which the ratio τ_y/σ_T is in between $0.54 \leq \tau_y/\sigma_T \leq 0.62$ (see Eq. (7)). Note that for $c = 0$, $\tau_y/\sigma_T = 1/\sqrt{3}$ *and* the Drucker criterion reduces to the von Mises criterion. The yield surfaces for isotropic materials with their yielding behavior characterized by $c > 0$ (i.e. $\tau_y/\sigma_T < 1/\sqrt{3}$) are interior to the von Mises one, while for isotropic materials characterized by

$c < 0$ (i.e. $\tau_y/\sigma_T > 1/\sqrt{3}$), their yield loci are exterior to the von Mises one. As an illustration, the projections of the yield loci in the biaxial plane $(\sigma_{xx}, \sigma_{yy})$ for materials characterized by $\tau_y/\sigma_T = 0.55$ ($c = 1.5$), $\tau_y/\sigma_T = 1/\sqrt{3}$ ($c = 0$, von Mises material) and $\tau_y/\sigma_T = 0.60$ ($c = -2$) are plotted in Fig. 1. The isotropic hardening of the material is characterized by a Swift hardening law, defined as

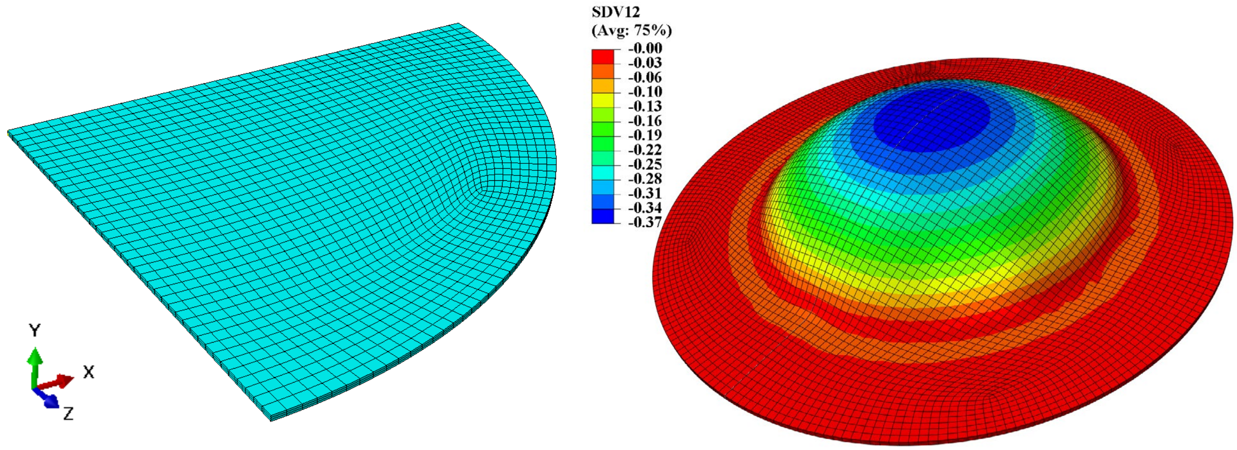
$$Y(\bar{\epsilon}^p) = K_0(\epsilon_0 + \bar{\epsilon}^p)^n \tag{8}$$

Where K_0, ϵ_0, and n are material parameters.

F.E. results for hemispheric and elliptic bulge tests

The main goal of our study is to investigate how yielding behavior characteristics of isotropic materials influence the finite element (F.E.) predictions of the distribution of plastic strain and thickness reduction in a bulge specimen for both hemispheric and elliptic die geometries. For this purpose, numerical simulations of the bulge test were conducted with our in-house UMAT developed for the constitutive model given by the Eq. (1)-(8). Due to the materials' isotropy and the symmetry of the bulging process, only one-quarter of the blank is meshed using 4557 hexahedral element with reduced integration (Abaqus C3D8R elements [8]). The same type of mesh for the blank was used for both hemispherical and elliptical bulge tests (see also Fig. 2(a)), i.e. only the geometries of the die was changed. The blank radius is of 100 mm and the blank's thickness is 1.8 mm, the drawbeads are modeled by radially pinning the nodes at the boundary of the blank. The applied bulging pressure is evolving linearly with time. The values of the elastic parameters considered were: Young's modulus, E=110 GPa and Poisson ratio, ν= 0.3. The parameters in the hardening law given by Eq.(8) were set to K_0=1050 MPa, ϵ_0= 0.001 and n= 0.2. As an example, in Fig. 2(b) is shown the F.E. prediction of the isocontours of the thickness plastic strain for a hemispherical bulge test of a material characterized by a stress ratio $\tau_y/\sigma_T = 0.55$ ($c = 1.5$) for a bulge height of 34.6 mm.

Simulation results are presented for both hemispheric and elliptic bulge test of three isotropic materials characterized by characterized by $\tau_y/\sigma_T = 0.55$ ($c = 1.5$), $\tau_y/\sigma_T = 1/\sqrt{3}$ ($c = 0$, von Mises material) and $\tau_y/\sigma_T = 0.60$ ($c = -2$) and both hemispherical and elliptical dies. The predicted thickness at the pole vs. height of the apex for the hemispheric bulging of the three isotropic materials is shown in Fig. 3. Note that even for hemispherical bulging, for which all the materials have the same yield stress under equibiaxial stretching (see also Fig.1), it is predicted that the thickness at the pole depends on the yielding ratio τ_y/σ_T of the material. For a same bulge height, the lower is the value of the ratio τ_y/σ_T, the lower is the thickness reduction. As an example, for the apex experienced a thickness reduction of 25%, it is predicted that the bulge height for a von Mises material is of 31.93 mm, while for a material characterized by $\tau_y/\sigma_T = 0.55$ ($c = 1.5$), the bulge height is of 33.24 mm and for an isotropic material characterized by $\tau_y/\sigma_T = 0.60$ ($c = -2$), the predicted bulge height is of 30.67 mm. This suggests that for a material characterized by a ratio $\tau_y/\sigma_T > 1/\sqrt{3}$ (i.e. a material with a yield locus exterior to the von Mises one), neglecting the specificity of the yielding behavior and modeling its plastic behavior using the von Mises criterion could results into underestimating the thinning of the material under hemispherical bulging.



(a) (b)

Fig. 2. (a) F.E. mesh of the specimen used for the bulge test with circular die.(b) Hemispherical bulging: F.E. predictions of the isocontours of thickness strain for a material by characterized by $\tau_y/\sigma_T = 0.55$.

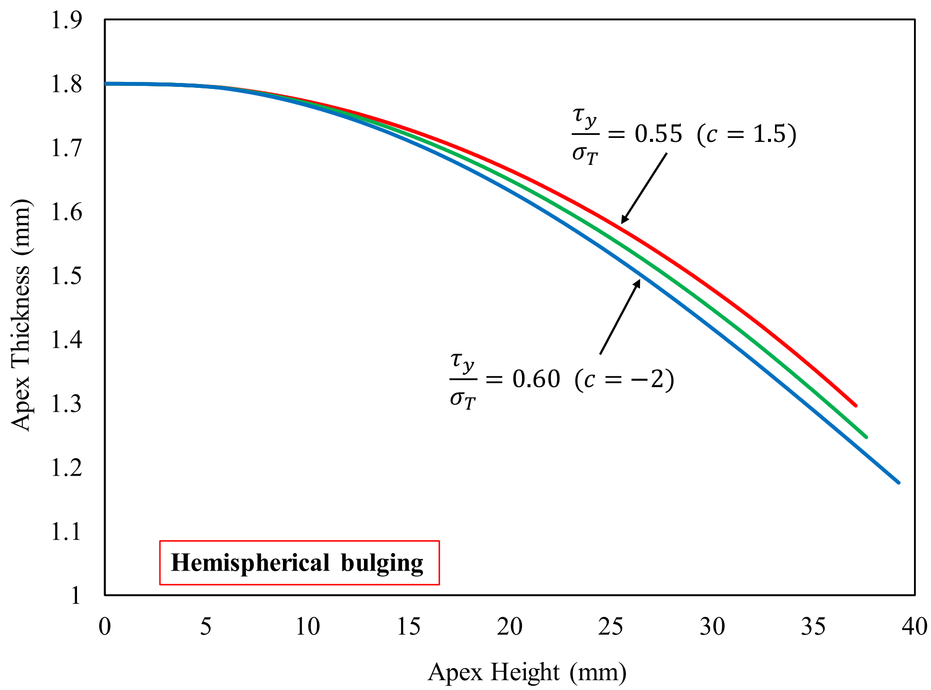


Fig. 3. F.E. Predictions according to the Drucker [7] yield criterion of the thickness vs. height at the apex of the hemispheric bulge for isotropic materials having various ratios between the yield in shear and uniaxial tension $\tau_y/\sigma_T = 0.55$ *(*$c = 1.5$*),* $\tau_y/\sigma_T = 1/\sqrt{3}$ *(*$c = 0$*, von Mises material) and* $\tau_y/\sigma_T = 0.60$ *(*$c = -2$*).*

Same conclusions could be drawn from the predicted thickness at the pole vs. height of the apex for elliptical bulging, i.e for a same height of the apex, the thinning will be delay for a material characterized by a ratio $\tau_y/\sigma_T < 1/\sqrt{3}$ compare to a von Mises material. On the other hand, thinning will be occurring for a lower apex height for a material characterized by a ratio $\tau_y/\sigma_T > 1/\sqrt{3}$ than a von Mises material. As an example, Fig. 4 shows the F.E. results according to the Drucker yield criterion for the thickness vs. height at the apex of the elliptic bulge with die ratio $a/b = 1.29$ (*a* and *b* denoting the long axis and short axis of the elliptic die, respectivelly) for three isotropic materials. It is worth mentioning the 5.1 % difference in bulge height between the considered isotropic materials to reach the same thickness reduction (25% thickness reduction).

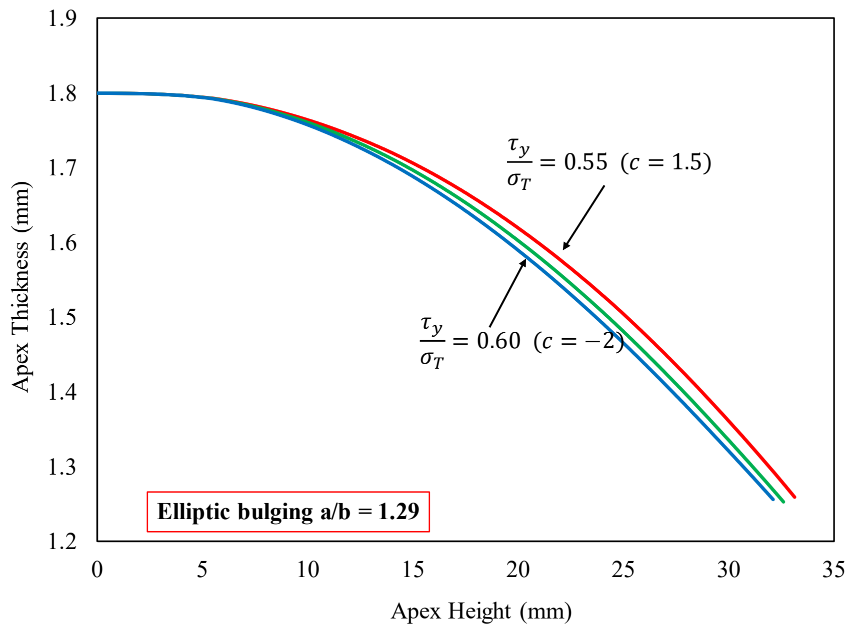


Fig. 4. F.E. Predictions according to the Drucker [7] yield criterion of the thickness vs. height at the apex of the elliptic bulge (die ratio a/b = 1.29) for isotropic materials having various ratios between the yield in shear and uniaxial tension τ_y/σ_T *=0.55 (c= 1.5),* $\tau_y/\sigma_T = 1/\sqrt{3}$ *($c = 0$, von Mises material) and* $\tau_y/\sigma_T = 0.60$ *($c = -2$).*

It is worth further examining the influence of the material yielding characteristics on the predicted ratio between the two in-plane strains developing at the apex of the bulge. As expected, under hemispherical bulging, irrespective of the material, the strains ratio is about 1 (see also the yield loci plotted in Fig. 1). However, for an elliptic bulge test, the strain paths achieved during the same experiment are strongly dependent on the plastic properties of the material, namely the ratio τ_y/σ_T. Let denote major strain by the strain developing in the direction of the short axis of the elliptic die and minor strain, the strain occurring in the direction of the long axis of the elliptic die.

As an example, it is shown in Fig. 5 the evolution of the major to minor plastic strain ratio developing in an elliptic bulging test with an die aspect ratio of 1.29 for three isotropic materials. The F.E. results indicated that for the same forming process, the strain ratio occurring at the apex of the elliptic bulge is dependent on the yielding properties of the material. For a die aspect ratio of 1.29, the plastic strain ratio is 1.37 for the von Mises material, 1.42 for the material characterized by $\tau_y/\sigma_T = 0.55$ and 1.29 for a material characterized by $\tau_y/\sigma_T = 0.60$. Therefore, for the same bulge test, the difference in strain ratio between the three isotropic materials is about ~10%.

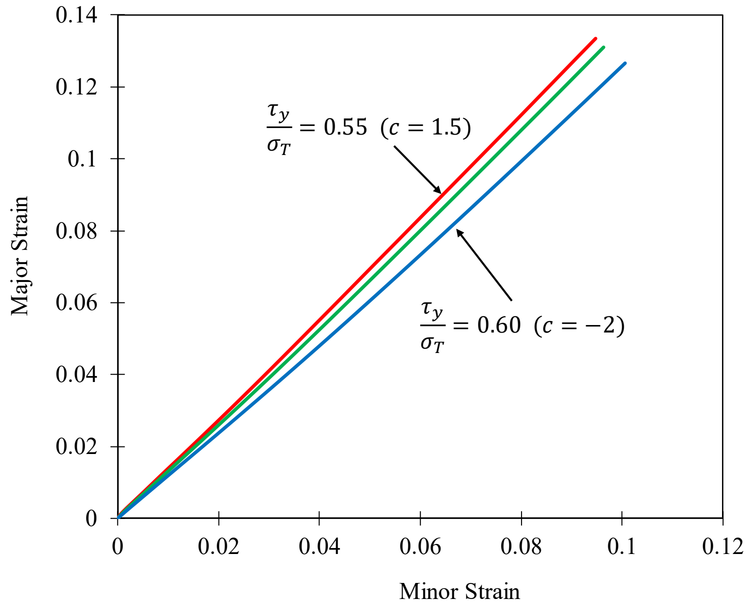


Fig. 5. Influence of the yielding ratio τ_y/σ_T on the major to minor plastic strains ratio at the pole for isotropic materials according to Drucker [7] and von Mises criterion for materials with various ratios between the yield in shear and uniaxial tension $\tau_y/\sigma_T = 0.55$ ($c = 1.5$), $\tau_y/\sigma_T = 1/\sqrt{3}$ ($c = 0$, von Mises material) and $\tau_y/\sigma_T = 0.60$ ($c = -2$).

Conclusions

A study of the influence of the ratio between the yield stress in uniaxial tension and in pure shear on the behavior under bulging of isotropic materials has been performed. F.E. simulations of bulge tests were conducted to study how the specificities of the plastic behavior of an isotropic material influences the strain paths and stress paths experienced during elliptical bulging. The plastic behavior of the isotropic material was described by the Drucker's [7] yield criterion. This yield criterion involves a unique parameter c, which is solely expressible in terms of the ratio between the yield stress in simple tension and that in shear τ_y/σ_T. F.E. simulations of bulge tests were performed for various isotropic materials with different ratios τ_y/σ_T to study how the plastic behavior of an isotropic material influences the strain paths and stress paths experienced during biaxial stretching. The F.E. results show that for both hemispheric and elliptic bulge tests, the greatest thickness reduction is predicted for a material characterized by a yield locus exterior to the von Mises yield surface (or $\tau_y/\sigma_T > 1/\sqrt{3}$) while for material whose yield surface is interior to the von Mises one, a lowest thickness reduction is predicted. Moreover, for elliptical bulging, the plastic strain ratio achieved during the bulge test is larger for a material characterized by $\tau_y/\sigma_T < 1/\sqrt{3}$ ($c > 0$) than in the case of a von Mises material. On the other hand, for a material characterized by a yielding ratio $\tau_y/\sigma_T > 1/\sqrt{3}$ ($c < 0$), the plastic strain ratio is less than in the case of a von Mises material.

F.E. simulations of bulge tests were conducted to study how the specificities of the plastic behavior of an isotropic material influences the strain paths and stress paths experienced during elliptical bulging.

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