

## Effect of Experimental Determination Process on Shear Stress Coefficient of Green Equation Describing HIP

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**Keywords:** HIP Modeling, HIP Tooling Design

**Abstract.** Despite the uniformity of the boundary conditions on the surface of the HIP capsule during the HIP Cycle (Pressure, Temperature) and full uniformity of stresses inside the powder material at the end of the cycle (isotropic tensor), the compressive deformation of the capsules with powder has a substantial shear component that can be described as distortion or non-uniformity. Accounting the values of shear deformation is the subject of HIP modeling and HIP tooling design. To determine the functions that adequately describe deformation of capsules with the given powder material during HIP, it is important to carry out experiments so that the values of the deformation tensor are close to those in the powder during a real HIP process. The adequacy of 3 deformation conditions for a combination TA6-4 powder and 304L capsule was evaluated for the database generation: HIP in capsules with different stiffness (deformation close to flat); Uni-directional upsetting of porous samples; Upsetting in a rigid die (uni-direction deformation). After homogenization of full dense material and capsule material, coefficients of Green's equation have been calculated for TA6-4 powder in a 304L capsule. Unidirectional densification and HIP dilatometer give similar results when upsetting results are apart. Analysis of results shows that, in actual conditions of HIP cycle, only a small portion of ellipsoidal plasticity surface given by Green's equation is used. Unidirectional pressing and HIP dilatometer condition are on the actual portion of ellipse. Upsetting of porous material is out of the useful zone and seems not convenient to determine shear stress coefficients.

### Introduction

To determine the functions that adequately describe deformation of capsules with the given powder material during HIP, it is important to carry out experiments so that the values of the deformation tensor are close to those in the powder during a real HIP process.

Analysis of results shows that, in actual conditions of HIP cycle, only a small portion of ellipsoidal plasticity surface given by Green's equation is used for the actual stress-strain conditions:

1. Deformation of porous media and plasticity surface
2. At every elementary increment of temperature and pressure the powder material is moving to a new plasticity surface
3. Plasticity surface is presented as a set of changing with ellipsoidal surfaces in the space of the main stresses (1-1-1) and deformation of the material
4. Occurs in some vicinity of the axis, on the "cap" of the ellipsoid



To describe the behavior of the powder material the elliptical Green's plasticity criterion was used:

$$\frac{\sigma^2}{f_2^2} + \frac{s^2}{f_1^2} = Y^2. \tag{1}$$

where:  $\sigma = \frac{1}{3}(\sigma_r + \sigma_\phi + \sigma_z)$  - the average stress ( $\sigma_{ij}$  - the corresponding components of the stress tensor);

$$s^2 = \frac{3}{2} \left[ \frac{(2\sigma_r - \sigma_\phi - \sigma_z)^2}{9} + \frac{(2\sigma_\phi - \sigma_r - \sigma_z)^2}{9} + \frac{(2\sigma_z - \sigma_r - \sigma_\phi)^2}{9} + 2\sigma_{r\phi}^2 + 2\sigma_{rz}^2 + 2\sigma_{\phi z}^2 \right] -$$

intensity of the stress tensor deviator;  $f_1, f_2$  - functions of powder density  $\rho$  defined from experiments;  $Y$  - yield strength of the monolithic material.

*“Classical” Experiments to define the functions  $f_1, f_2$ .*

First experiment-HIP in a thin-walled capsule.

If we ignore the influence of the capsule walls, and  $\sigma_r = \sigma_\phi = \sigma_z = -P$

Then from relation (1), we get:  $\frac{P^2}{f_2^2} = Y^2 \Rightarrow f_2^2 = \frac{P^2}{Y^2} \Rightarrow f_2 = \frac{P}{Y}$ .

Second experiment- upsetting of a cylindrical porous sample;  $\sigma_r = \sigma_\phi = 0, \sigma_z = -T$

From relation (1), we get:  $\frac{T^2}{9f_2^2} + \frac{T^2}{f_1^2} = Y^2$ .

As far as  $\frac{1}{f_2^2} = \frac{Y^2}{P^2}$ , then  $\frac{1}{f_1^2} = \frac{Y^2}{T^2} \left( 1 - \frac{T^2}{9P^2} \right)$

The vector of stresses in the principal axes is  $\{0; 0; -T\}$ . The hydrostatic axis has a direction  $\{-1; -1; -1\}$  and the angle between them is  $\psi$ . Then  $\cos\psi = 1 / \sqrt{3} = 0.577$ .  $\psi \approx 0.95 \approx 54^\circ$ .

This means that free upsetting of porous samples occurs at the angle of 54 degrees from the hydrostatic axis.

Now let us explore the real stress conditions of the HIP process. For the strain rates during deformation of powder material we have:

$$\varepsilon_{ij} = \omega \frac{\partial \Phi}{\partial \sigma_{ij}}. \tag{2}$$

where: -  $\varepsilon_{ij}$  - the corresponding components of the strain rate tensor; and  $\Phi(\sigma_{ij}) = 0$  equation for the flow surface.

Let us consider that  $\varepsilon_r, \varepsilon_\phi, \varepsilon_z$  - main components of the strain rate tensor. In the majority of the HIP processes (can geometries) these components are compressive,

Let us designate  $\frac{\varepsilon_\varphi}{\varepsilon_r} = \alpha$   $\frac{\varepsilon_z}{\varepsilon_r} = \beta$  and consider  $0 \leq \beta \leq \alpha \leq 1$ . Using (1, 2), we can define the important value of the ANGLE between the “vector of stresses” and hydrostatic axis.

$$\cos \Psi = \frac{1}{\sqrt{3}} \times \frac{\{27f_2^2\}}{\sqrt{\left[9f_2^2 + 2f_1^2 \frac{(2-\alpha-\beta)}{(1+\alpha+\beta)}\right]^2 + \left[9f_2^2 + 2f_1^2 \frac{(-1+2\alpha-\beta)}{(1+\alpha+\beta)}\right]^2 + \left[9f_2^2 + 2f_1^2 \frac{(-1-\alpha+2\beta)}{(1+\alpha+\beta)}\right]^2}} \quad (3)$$

Analyzing this relationship, it is possible to show that at  $0 \leq \beta \leq \alpha \leq 1$ :

$$\cos \Psi \geq \frac{\{9f_2^2\}}{\sqrt{81f_2^4 + 8f_1^4}} \quad (4)$$

In reality the value of  $f_1$  is usually less than  $f_2$ . Therefore

$$\cos \Psi \geq \frac{9}{\sqrt{89}} \approx 0.954. \text{ Then } \psi < 0.31 \approx 18^\circ. \quad (5)$$

This means that in the real HIP conditions the angle  $\psi$  is less than 18 degrees.

In order to be in the real stress condition compatible with HIP, the second experiment for the definition of  $f_1$  and  $f_2$  is better to conduct as uni-directional deformation of powder in a rigid die.

Considering that  $\sigma_z = -F$  - is a known value and.  $\varepsilon_r = \varepsilon_\varphi = 0$

Then:

$$\sigma_z = \frac{1}{\omega} \left[ \left( \frac{9f_2^2 - 2f_1^2}{18} \right) \varepsilon_r + \left( \frac{9f_2^2 - 2f_1^2}{18} \right) \varepsilon_\varphi + \left( \frac{9f_2^2 + 4f_1^2}{18} \right) \varepsilon_z \right] \quad (6)$$

$$\frac{1}{\omega} = \frac{6Y}{\sqrt{(9f_2^2 - 2f_1^2)(\varepsilon_r + \varepsilon_\varphi + \varepsilon_z)^2 + 6f_1^2(\varepsilon_r^2 + \varepsilon_\varphi^2 + \varepsilon_z^2 + 2\varepsilon_{r\varphi}^2 + 2\varepsilon_{\varphi z}^2 + 2\varepsilon_{rz}^2)}} \quad (7)$$

To

$$\sigma_z = \frac{6Y}{\sqrt{(9f_2^2 - 2f_1^2)(\varepsilon_r + \varepsilon_\varphi + \varepsilon_z)^2 + 6f_1^2(\varepsilon_r^2 + \varepsilon_\varphi^2 + \varepsilon_z^2 + 2\varepsilon_{r\varphi}^2 + 2\varepsilon_{\varphi z}^2 + 2\varepsilon_{rz}^2)}} \times \left[ \left( \frac{9f_2^2 - 2f_1^2}{18} \right) \varepsilon_r + \left( \frac{9f_2^2 - 2f_1^2}{18} \right) \varepsilon_\varphi + \left( \frac{9f_2^2 + 4f_1^2}{18} \right) \varepsilon_z \right] \quad (8)$$

$$\sigma_z = \frac{6Y}{\sqrt{(9f_2^2 - 2f_1^2)\varepsilon_z^2 + 6f_1^2\varepsilon_z^2}} \left[ \left( \frac{9f_2^2 + 4f_1^2}{18} \right) \varepsilon_z \right] \quad (9)$$

$$\sigma_z = -\frac{Y(9f_2^2 + 4f_1^2)}{3\sqrt{9f_2^2 + 4f_1^2}} \quad (10)$$

$$F = \frac{Y}{3} \sqrt{9f_2^2 + 4f_1^2} \quad (11)$$

$$\frac{9F^2}{Y^2} = 9f_2^2 + 4f_1^2 \quad (12)$$

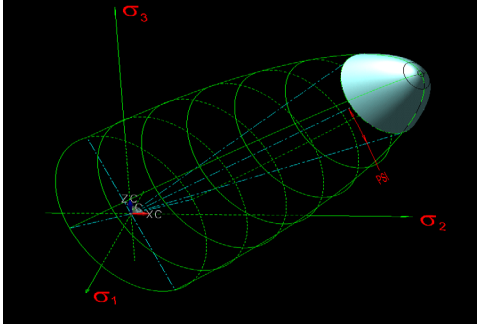
As  $f_2^2 = \frac{P^2}{Y^2}$ , then

$$\frac{9F^2}{Y^2} = 9\frac{P^2}{Y^2} + 4f_1^2 \Rightarrow 9\left(\frac{F^2}{Y^2} - \frac{P^2}{Y^2}\right) = 4f_1^2 \quad (13)$$

$$f_1^2 = \frac{9}{4} \frac{F^2 - P^2}{Y^2} \Rightarrow f_1 = \frac{3}{2Y} \sqrt{F^2 - P^2} \quad (14)$$

We can then build the following Table of Angles on the Plasticity Surface validating the experimental techniques for defining the  $f_1$  and  $f_2$  plasticity functions.

Table 1. Angles on the Plasticity Surface for Different Processes

N	Condition	Angles on the Plasticity Surface for different processes, degrees 	Comments
1	Hydrostatic conditions (very thin or very soft can material)	0	1 <sup>st</sup> “classical” experiment, can work for high strength PM alloys, defines the function $f_2$
2	Free upsetting of porous samples obtained in the interrupted cycles	54	2 <sup>nd</sup> “Classical” experiment” to define $f_1$ , but the angle $\psi$ is far from HIP working area
3	Uni-directional compressing	18	Difficult to accomplish, needs a rigid die
4	“Flat” deformation, $\varepsilon_z = 0$	9.2	Valid for HIP of a long cylinder
5	Maximal angle during HIP	18	Can be reached when the capsule is very rigid in two directions and the deformation occurs mainly in one direction

### Conclusions

Despite the uniformity of the boundary conditions on the surface of the HIP capsule during HIP and full uniformity of stresses inside the powder material at the end of the cycle, the compressive deformation of the capsules with powder has a substantial shear component. Accounting these values of shear deformation is the subject of HIP modeling and HIP tooling design.

To determine the functions that adequately describe deformation of capsules with the given powder material during HIP, it is important to carry out experiments so that the values of the stress tensor are close to those in the powder during a real HIP process.

Theoretical analysis of results shows that, in actual conditions of HIP cycle, only a small portion of ellipsoidal plasticity surface in the space of main stresses given by Green’s equation (the “cap”) is used.

Unidirectional pressing, flat deformation and HIP dilatometer condition are on the actual portion of ellipsoidal surface. Upsetting of porous material is out of the useful zone to determine shear stress coefficients.