A peridynamics elastoplastic model with isotropic and kinematic hardening for static problems

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Abstract. This study proposes a formulation equivalent to J2 plasticity with the associated flow rule to simulate the elastoplastic behavior of materials with isotropic or kinematic hardening in a peridynamic framework. The capabilities of the developed formulation are analysed through 2D and 3D case studies whose results (displacement and stress field) are compared with those obtained from the corresponding FEM models.

Introduction

The evaluation of the structural residual life of aerospace structures requires the ability to predict damage propagation. In recent years, Peridynamics (PD) [1], a new non-local continuum theory, attracted the attention of many researchers for its capability to simulate crack initiation, propagation and interaction. The theory has been widely used to model crack propagation in brittle materials, while the analysis of elastoplastic materials [2] has been mainly limited to the perfectly elastoplastic behavior [3,4]. Unfortunately, in the case of metals, experimental observations reveal a complex plastic behavior, which requires models with isotropic and kinematic hardening [2]. A PD constitutive model for 2D elastoplasticity with isotropic hardening is presented in [6] and extended to the 3D case in [7]. In this paper, an elastoplastic formulation equivalent to J2 plasticity is presented with which it is possible to simulate the elastoplastic behavior in the case of both isotropic and kinematic hardening.

Formulation

The PD equation of motion, for the static case, is [1]:

$$\int_{\mathcal{H}} \left(\underline{T}[\boldsymbol{x}]\langle \boldsymbol{x}' - \boldsymbol{x}\rangle - \underline{T}[\boldsymbol{x}']\langle \boldsymbol{x} - \boldsymbol{x}'\rangle\right) dV_{\boldsymbol{x}'} + \boldsymbol{b}(\boldsymbol{x}) = 0 \tag{1}$$

where $\boldsymbol{b}(\boldsymbol{x})$ is the body force on the material point $\boldsymbol{x}$, $\underline{T}[\boldsymbol{x}]$ is the force state on the material point $\boldsymbol{x}$ corresponding to the bond vector $\boldsymbol{x}' - \boldsymbol{x}$ and $\mathcal{H}$ represents the neighborhood of $\boldsymbol{x}$ whose radius δ is the *horizon,* see Fig.1. The force vector state is a vector of modulus $\underline{t}$ (the scalar force state) and direction coincident with the deformed configuration of the bond (ordinary state-based PD).

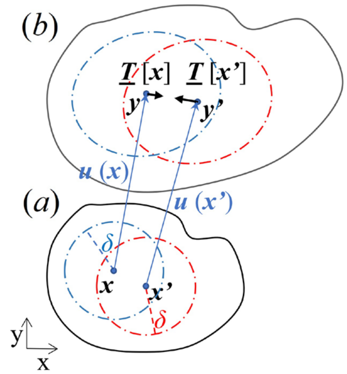

Figure 1: Positions of two interacting material points $\boldsymbol{x}$ *and* $\boldsymbol{x}'$*; (a) initial and (b) deformed configuration.*

The extension of a bond (a scalar quantity) is $\underline{e} = \underline{y} - \underline{x}$, where $\underline{y} = |\boldsymbol{y}' - \boldsymbol{y}|$ and $\underline{x} = |\boldsymbol{x}' - \boldsymbol{x}|$. $\boldsymbol{x}$ is the position of the material point in the initial configuration while $\boldsymbol{x}'$ denotes the generic point belonging to the neighborhood of $\boldsymbol{x}$. While $\boldsymbol{y}$ and $\boldsymbol{y}'$ are, respectively, the positions of $\boldsymbol{x}$ and $\boldsymbol{x}'$ in the deformed configuration.

In order to study problems involving elastoplastic behavior, it is necessary to distinguish between the isotropic and deviatoric component of the bond extension and of the scalar force state [1]. The extension of a bond is the sum of two components [1]: the isotropic and the deviatoric. Therefore, $\underline{e} = \underline{e}^{iso} + \underline{e}^{d}$, similarly for the scalar force state we have: $\underline{t} = \underline{t}^{iso} + \underline{t}^{d}$. In [3,4] it is emphasized that $\underline{t}^{iso}$ does not depend on $\underline{e}^{d}$ and the extension component $\underline{e}^{iso}$ is only elastic. Whereas $\underline{e}^{d}$ is itself the sum of two components, elastic and plastic respectively $\underline{e}^{d} = \underline{e}^{de} + \underline{e}^{dp}$. Finally, the deviatoric component of the scalar force state, in the case of materials with elastoplastic behavior, results in the 3D case, [4]:

$$\underline{t}^{d} = -5\mu\theta \frac{\underline{\omega x}}{m} + \frac{15\mu}{m} \underline{\omega}(\underline{e} - \underline{e}^{dp}) \tag{3}$$

Where μ is the shear modulus, θ is the dilatation, m is the weighted volume and $\underline{\omega}$ is the influence function (see [1,4] for further details).

Furthermore, on the basis of classical plasticity theory, the load-unload conditions in the Kuhn-Tucker form and the consistency condition [2] must be fulfilled when solving elastoplastic problems. In the case of the peridynamic formulation [3] these conditions become:

$$\begin{cases} \lambda \geq 0, \ f(\underline{t}^{d}) \leq 0, \quad \lambda f(\underline{t}^{d}) = 0 \\ \qquad \lambda \dot{f}(\underline{t}^{d}) = 0 \end{cases} \tag{4}$$

where f is the yield function and λ is the continuum consistency parameter; while the plastic flow rule is [3]:

$$\dot{\underline{e}}^{dp} = \lambda \nabla^{d} \psi(\underline{t}^{d}) \tag{5}$$

in which $\nabla^{d} \psi(\underline{t}^{d})$ is the constrained Fréchet derivative of $\psi(\underline{t}^{d})$ defined hereafter.

In [3,4] based on the formulation introduced in [1] the following equation for the yield function for materials with perfectly plastic behavior is proposed:

$$f(\underline{t}^d) = \psi(\underline{t}^d) - \psi_0 = \frac{\|\underline{t}^d\|^2}{2} - \psi_0 \tag{6}$$

where $\|\underline{t}^d\|^2 = \int_{\mathcal{H}} (\underline{t}^d)^2 dV_{x'}$ and $\psi_0 = 25\sigma_Y^2/8\pi\delta^5$ in which σ_Y is the material's yield stress. Eq.6 is equivalent to the yield function $f = \sigma_{\mathrm{vM}} - \sigma_Y$ used in classical mechanics [2] where σ_{vM} is the von Mises stress.

The proposed formulation to study the behavior of materials with isotropic and kinematic hardening, was inspired by the corresponding yield function used in classical mechanics [2]

$$f = |\sigma_{\mathrm{vM}} - q| - (\sigma_Y + K\alpha) \tag{7}$$

In this equation, K is the isotropic hardening modulus, q is the back stress resulting from the kinematic hardening, and α is the internal hardening variable. q and α, initially zero, vary according to the following equations [2]:

$$\dot{q} = \dot{\varepsilon}_p H \quad \text{and} \quad \dot{\alpha} = \mathrm{sign}(\sigma_{VM} - q)\dot{\varepsilon}_p \tag{8}$$

Where H is the kinematic hardening modulus and ε_p is the equivalent plastic strain. Rewriting Eq. 7 in the context of the peridynamic formulation (for details see [5]) one obtains an equation analogous to Eq.6 in which, however, ψ_0 is no longer a constant and depends on the load increment at the material point considered. Therefore ψ_0 becomes:

$$\psi_0(\boldsymbol{x}, t) = \frac{25[\sigma_Y + K\alpha + \mathrm{sign}(\sigma_{\mathrm{vM}} - q)q]^2}{8\pi\delta^5} \tag{9}$$

In Eq.9, q and α are found using Eq.8 in which the equivalent plastic strain should be replaced with the corresponding quantity expressed in the peridynamic formulation, which is a function of the deviatoric plastic extension [6,7]. It is worth noting that Eq.9 is obtained by considering small displacements.

The numerical implementation strategy of the proposed formulation involves discretizing the domain with a uniform grid of nodes. Then the non-linear static problem is solved using an incremental approach. Therefore, an iterative procedure using a return mapping algorithm was implemented for the determination of the deviatoric plastic extension and the various dependent quantities [5].

Numerical examples

In the following examples E=200 (GPa), ν=0.3, ρ=8000 (kg/m^3) and σ_{y0}=600 (MPa). The isotropic hardening modulus is K=20 (GPa), and the kinematic hardening modulus is H=20 (GPa). All cases were studied using the proposed PD formulation and classical FE simulations.

The first case is a thin plate with a central hole (Fig.2a) in plane stress conditions, subjected to an imposed displacement (Fig.2b) applied in increments of 0.0125 (mm).

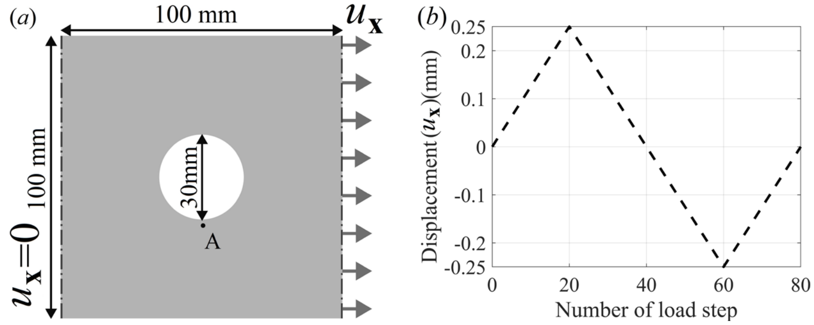

Figure 2: (a) Geometry and boundary conditions; (b) displacement loading in the x direction.

In Fig. 3, the displacements in the *x*-direction calculated by PD and FE at the 20th load step (u_x=0.25 mm) are compared. Good agreement can be observed between the results obtained from the two models.

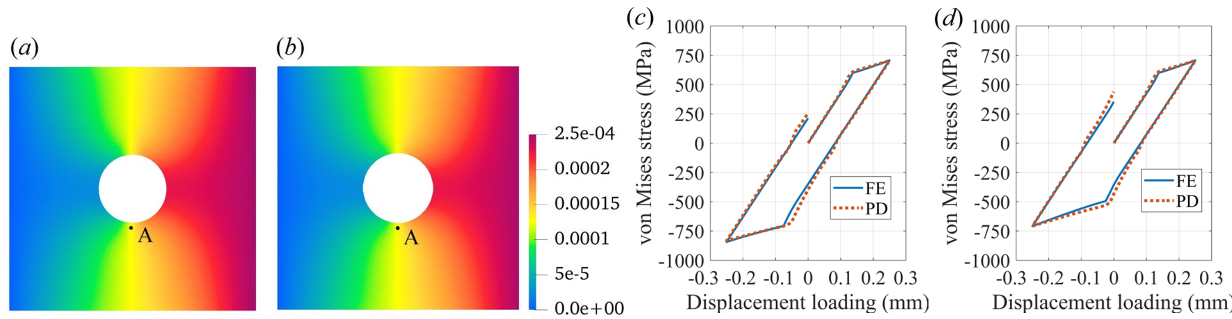

Figure 3: Displacement (m) in the x direction solved by: (a) FE, (b) PD. von Mises stress at point A vs loading displacement, (c) isotropic hardening case; (d) kinematic hardening case.

Fig.3c-d compares the von Mises stress at point A (Fig.2a) for the entire load history for both the isotropic (Fig.3c) and kinematic (Fig.3d) hardening cases. Point A is located in the region of the body where plastic deformation develops. The agreement between the PD and FE results is good: in particular, the PD model correctly estimates the maximum expected stress values obtained with the FE model. Note that in Fig.3c-d positive von Mises stresses are associated with a tensile load and negative von Mises stresses with a compressive load.

The second example studies a 3D structure whose dimensions, load and constraint conditions are shown in Fig.4. The material behavior is elastoplastic with isotropic hardening. The load cycle (imposed displacement) is similar to that shown in Fig.2b with a maximum displacement value of 0.35 (mm). Fig.4 compares the results obtained with the PD and FE models in terms of both displacement (u_y component) and von Mises stress in the *xy* plane containing the longitudinal axis of the specimen at the 20th load increment (at which the maximum displacement is applied). Good agreement is observed between the PD and FEM results despite the fact that no strategies were adopted to mitigate the surface effect in the PD model.

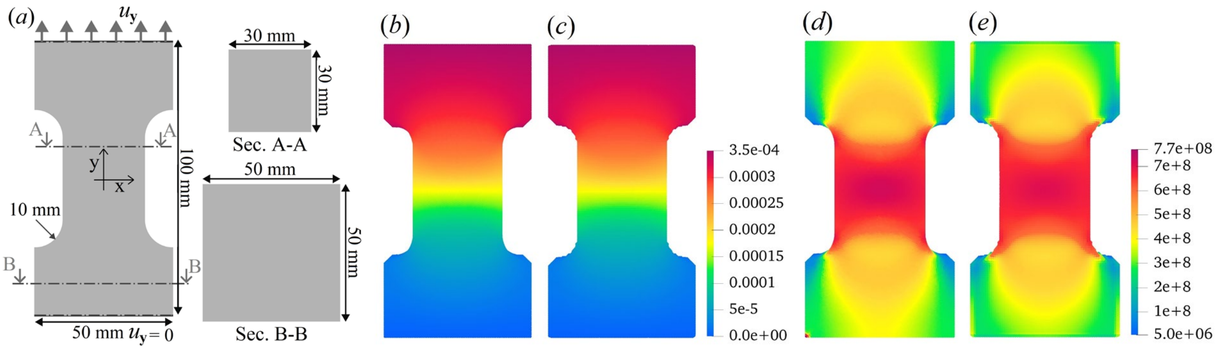

Figure 4: 3D example, a) main dimensions. Displacement (m) in the y direction on plane xy computed by: (b) FE, (c) PD ($u_y = 3.5 \cdot 10^{-4}$ m). Distribution of von Mises stress (Pa) on plane xy obtained by: (d) FE, (e) PD ($u_y = 3.5 \cdot 10^{-4}$ m).

Conclusions

This study presents an extension of the elastoplastic model in Peridynamics capable of describing isotropic and kinematic hardening behavior. The proposed formulation is equivalent to J2 plasticity with associated flow rule. 2D and 3D cases were studied, and the comparison between the results of the PD models and the corresponding FE models highlighted the capabilities of the developed approach, which represents the necessary first step for the simulation of ductile fracture in a peridynamic framework.

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