Thermal buckling analysis and optimization of VAT structures via layer-wise models

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Abstract. The present study investigates the combination of different manufacturing parameters, such as the curvature radius of the single fiber along its path on a symmetric stacking sequence of the Variable Angle Tows composite (VAT). Moreover, the study objective is to individuate the VAT configuration to maximize the critical thermal buckling load of a thermal-loaded composite square plate. Numerical simulations are performed in the Carrera Unified Formulation (CUF) framework, which allows for a high-order two-dimensional (2D) theory based on the Finite Element Method (FEM), enabling the Layer Wise (LW) discretization of the model. The linearized buckling problem is involved in the formulation, and the resolution of the eigenvalue problem leads to finding the thermal buckling critical temperature.

Introduction

Composite materials have superior strength and stiffness properties compared to traditional materials but are often influenced by the environment. Those involving an increase in material temperature induce expansion strains, and consequently, the buckling critical load can be dramatically affected by an over-temperature. The thermal environment is particularly relevant for space applications with a consistent radiation heating. Instead, for high-speed aeronautical applications, the heating is usually imposed on the structures by the drag. Due to the applied overtemperature, buckling deflection may occur suddenly under specific load conditions, and the deformation of the plate can significantly influence the structure behavior. If a constant along the thickness temperature profile is applied, deflection occurs only at the unique critical temperature.
Variable Angle Tows (VAT) materials exploit a new manufacturing technology introducing an additional degree of freedom in the fiber deposition that can follow curved shapes [1]. It is well known that VAT laminates can improve buckling performance compared to classical composites [2]. Furthermore, thermal buckling has been deeply studied for all the materials that can be subjected to thermal environments [3]. The present work focuses on the fiber deposition optimization of a thermally-loaded square plate to retard the thermal buckling phenomenon. The plate theory is applied within the Carrera Unified Formulation (CUF) framework combined with the Finite Element Method (FEM) approximation [4]. Different kinematic theories are employed, and the results are compared and discussed.

Model description

The decoupled approach is employed to describe the thermal problem. The primary variables of the problem are the displacements, denoted by $\boldsymbol{u}^T$:

$$\boldsymbol{u}^T(x, y, z) = (u_x, u_y, u_z) \tag{1}$$

where the symbol T denotes transposition. As reported in Eq. (2), the strains $\boldsymbol{\varepsilon}$ can be obtained from the displacement by the geometrical relations using the non-linear differential operator b. The stresses $\boldsymbol{\sigma}$ are calculated from the strains through the Hooke's law, by applying the material properties matrix C as in Eq. (3). Note that in the VAT case, the matrix C depends on the local fiber orientation and is not globally defined as in the case of classical configuration.

$$\boldsymbol{\varepsilon} = \left(\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xz}, \varepsilon_{yz}, \varepsilon_{xy}\right)^T = \boldsymbol{b}\,\boldsymbol{u} \tag{2}$$

$$\boldsymbol{\sigma} = \left(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xz}, \sigma_{yz}, \sigma_{xy}\right)^T = \boldsymbol{C}\,\boldsymbol{\varepsilon} \tag{3}$$

The CUF introduces an indicial notation where the three-dimensional displacements are divided into two components, the first $\boldsymbol{F}_\tau(z)$ denotes the kinematic expansion function, and the second $\boldsymbol{u}_\tau(x,y)$ is the in-plane unknown vector. Furthermore, the FEM is employed, and $\boldsymbol{u}_\tau(x,y)$ can be expressed by combining the shape functions $\boldsymbol{N}_i(x,y)$ and the nodal displacement vector $\boldsymbol{q}_{i\tau}$ as represented in Eq. (4).

$$\boldsymbol{u}(x,y,z) = \boldsymbol{F}_\tau(z)\,\boldsymbol{u}_\tau(x,y) = \boldsymbol{F}_\tau(z)\boldsymbol{N}_i(x,y)\,\boldsymbol{q}_{i\tau} \quad \tau = 1,2,\dots,M \quad i = 1,2,\dots,N_n \tag{4}$$

where the double index means sum, M is the number of expansion terms, and N_n is the number of nodes. Via the CUF, high-order theories are employed to describe the in-thickness behavior of the analyzed plate, and among the various theories, Taylor Expansion (TE) and Lagrange Expansion (LE) functions are selected as $\boldsymbol{F}_\tau(z)$ for the present investigation. Furthermore, the laminate properties are described with the Equivalent Single Layer (ESL) approach in the TE case and using the Layer Wise (LW) approach in LE models.

The CUF allows an invariant formulation of the problem using the Fundamental Nuclei (FN) that are not formally mutated with the employed expansion or on the number of nodes [4]. The indicial notation lets the problem governing equation building. In the case of buckling, using the PVD, as reported in Eq. (5), to obtain a linearized formulation of the governing equation based on the tangent stiffness matrix $\boldsymbol{K}_T$ is possible. Where $\boldsymbol{K}_T$ is expressed in terms of FN, the complete procedure can be found in [5].

$$\delta^2(L_{int}) = \int_V \delta(\delta\boldsymbol{\varepsilon}^T\boldsymbol{\sigma})dV = \delta\boldsymbol{q}_{sj}^T\,\boldsymbol{K}_T^{ij\tau s}\delta\boldsymbol{q}_{\tau i} = \delta\boldsymbol{q}_{sj}^T(\,\boldsymbol{K}_0^{ij\tau s} + \boldsymbol{K}_\sigma^{ij\tau s})\,\delta\boldsymbol{q}_{\tau i} \tag{5}$$

The buckling critical temperature is considered coincident with the bifurcation point, which is the load point where two equilibrium configurations exist. As a result, an eigenvalue problem must be solved to obtain the critical load:

$$|\boldsymbol{K_0} + \boldsymbol{\lambda_{cr}}\,\boldsymbol{K_\sigma}| = \boldsymbol{0} \tag{6}$$

Due to the linear approximation, the matrix $\boldsymbol{K}_\sigma$ is supposed to be proportional to λ_{cr}, which is the buckling critical load factor. $\boldsymbol{K}_\sigma$ is the geometric stiffness matrix strictly dependent on the internal pre-stress state due to the thermal over-temperature load.

Numerical results

In the present section, some numerical results about thermal buckling are presented. The VAT deposition angles are changed to obtain the best configuration retarding the buckling critical thermal load. Eq. (7) describes the fiber orientation θ. It depends on the three angles Φ, T_0, and T_1, which are, the reference system rotation angle, the fiber orientation in the center of the plate and,

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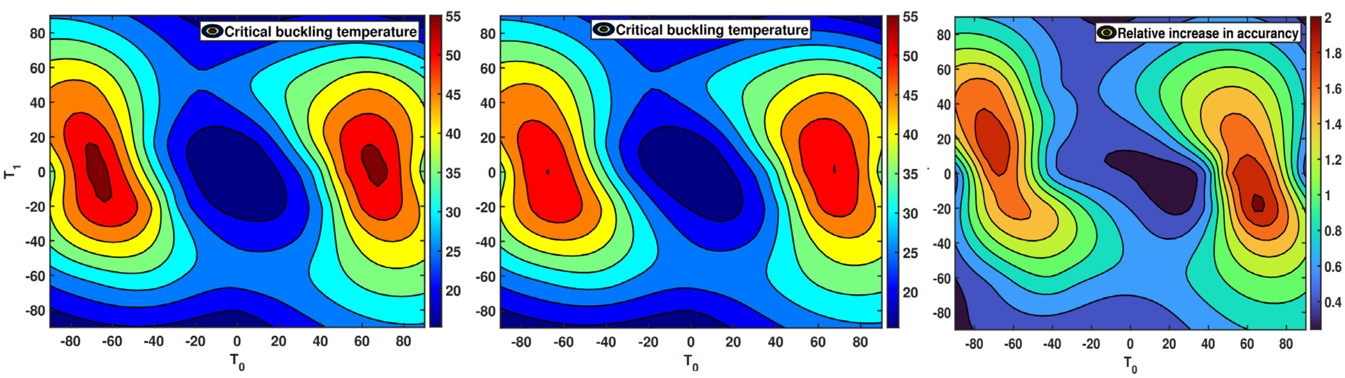
the angle at the edge, respectively. The symbol d denotes a characteristic distance, and x' is the new reference direction [1].

$$\theta = \Phi + T_0 + \frac{T_1 - T_0}{d} x' \tag{7}$$

For the sake of simplicity, only T_0 and T_1 are changed during the optimization with Φ equal to zero, the staking sequence is fixed at $[\theta/-\theta]_s$, and the plate is simply supported. The square plate has 150 mm edges and 1.016 mm thickness, and a one-degree constant over-temperature is applied on the whole plate. Two plates are analyzed, the first composed of Carbon/Epoxy whose properties are E_1 = 147 GPa, E_2 =10.3 GPa, ν_{12}=0.27, G_{12} = 7.0 GPa, α_1= -0.9 x 10^{-6} 1/K, α_2= 27.0 x 10^{-6} 1/K. The second plate is E-Glass/Epoxy with E_1 = 41 GPa, E_2 =10.04 GPa, ν=0.28 G_{12} = 4.3 GPa, α_1= 7.0 x 10^{-6} 1/K, α_2= 26.0 x 10^{-6} 1/K. Each plate presents a discretization of 20x20 Q9 in-plane elements, 2nd order TE and 3rd order LE are employed as expansion functions.



(a) TE2 *(b) LE3* *(c) Perc. difference*

Figure 1: (a), (b) Carbon/Epoxy critical buckling temperature varying the path angles, 20 x 20 Q9 in plane mesh. (a): Present method with ESL and TE2, (b) Present method with LW and LE3. (c) Percentage relative increasing accuracy from LE to TE2 ($\frac{\boldsymbol{\theta_{TE}-\theta_{LE}}}{\boldsymbol{\theta_{TE}}}\boldsymbol{x100}$). *Carbon/Epoxy.*

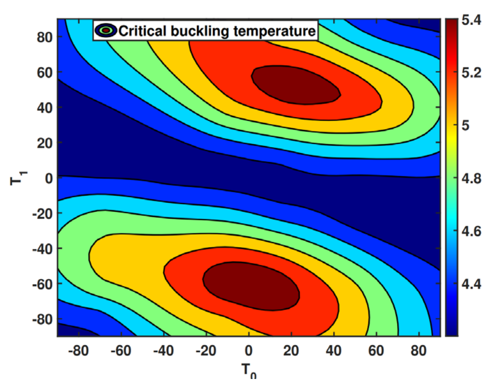


Figure 2: E-Glass/Epoxy critical buckling temperature varying the path angles, 20 x 20 Q9 in plane mesh, Present method with LW and LE3.

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Table 1:Maximum critical buckling temperature for VAT and straight configuration with $\boldsymbol{\theta} = \mathbf{45}°$.

	T_0	T_1	$\Delta\, T_{cr}$	$\Delta\, T_{straight}$
		Carbon/Epoxy		
20x20 Q9 LE3	-68.29°	0.45°	55.11°	41.32°
20x20 Q9 TE2	-67.39°	0.45°	56.27°	41.87°
Ref. [6]	69.00°	-5.71°	57.79°	42.21°
		E-Glass/Epoxy		
20x20 Q9 LE3	-4.97°	-60.15°	5.55°	5.38°
Ref. [6]	6.71°	58.04°	5.58°	5.43°

Conclusions

· As expected, the thermal buckling critical temperature graphs are symmetric to the line $T_0 = -T_1$ and present two maximum value zones for both configurations (Fig.1 and 2).
· Considering the adopted expansion theory, the 2nd-order TE model is more similar to the reference. Still, the LE model is more accurate even if it presents a greater computational cost. As depicted in Fig.1(c), the main difference is not in the critical temperature but mainly in the extension of the optimal zones. As a result, the zones with maximum temperature are abruptly reduced in the LE model reported in Fig.1(b) than in the less accurate model in Fig.1(a).
· Figure 1(c) clarifies that the increase in accuracy due to the LE model is not constant comparing different orientations angle but rises with the increase in the critical temperature.
· For the Carbon/Epoxy laminate, as collected in Table 1, the better path configuration allows reaching a critical over-temperature of 55.1° with a consistent gain to the straight deposition critical temperature of 41.3°.
· From Table 1 it is clear that E-Glass/Epoxy configuration does not present significant advantages using VAT deposition with a gain of 0.03% passing from a critical temperature of 5.384° to 5.55° in the VAT case.
· In the end, the choice of VAT deposition may be helpful to retard the buckling critical load, and, in Carbon/Epoxy case, the gain is more evident than in the Glass/Epoxy plate.

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