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Nonlinear mechanical analysis of aerospace shell structures through the discontinuous Galerkin method

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Abstract. The geometrically non-linear mechanical response of multilayer composite shells is addressed via an innovative discontinuous Galerkin formulation. In the framework of the Carrera Unified formulation, equivalent single layer kinematics with different through-the-thickness accuracy is adopted. The variational statement governing the shell nonlinear behavior is derived. The corresponding governing equations are solved via a discontinuous Galerkin approach, which employs the pure penalty method to weakly enforce the connection between the mesh elements. Numerical tests are presented to show the capabilities of the proposed approach.

Introduction

Multilayered composite shells are extensively employed as high-performance lightweight components in aerospace engineering. In advanced applications they may undergo large displacements, requiring non-linear analysis to characterize accurately their behavior. In this framework a fundamental role is played by the modelling and analysis of these structure that needs to be carried out with appropriate fidelity and cost effectiveness.

From the modelling point of view, besides fully three-dimensional models, shell structures are studied within the context of the so-called shell theories which can be classified into equivalent single layer (ESL) theories and layer-wise (LW) theories. Due to their complexity, the solution of shell theories models commonly requires the employment of numerical methods. The most common approach in the literature is the finite element method (e.g. Ref. [1]); the Ritz method [2], mesh-less solutions [3, 4] and the isogeometric analysis approach [5] have been also proposed.

Recently, the discontinuous Galerkin (dG) method emerged as a viable alternative showing interesting advantages in the use of nonstandard element and shape functions, in the application of non conformal meshes as well as high-order elements, in the implementation of meshing strategies such as hierarchical refinement and adaptativity and in scalable implementations [6, 7]. These features can underlie a robust treatment of complex problems as those involved in the analysis of multilayered shells [8, 9]. This motivate the present work in which high-order, equivalent single layer shell theories are solved through a pure penalty discontinuous Galerkin approach formulated accounting for geometrical nonlinear behavior [10].

Shell model and governing equations

In the context of a total Lagrangian approach , the shell deformation is described in terms of the displacement components expanded as [8, 9]

$$u_{\xi_i}(\xi_1,\xi_2,\xi_3)=\sum_{k=0}^{N_i} Z_k^i(\xi_3)U_{ik}(\xi_1,\xi_2) \tag{1}$$

where ξ_{m} are the curvilinear coordinates used to describe the shell geometry, N_i is the order of the assumed expansion, $Z_k^i(\xi_3)$ is the prescribed k-th function of the expansion and $U_{ik}(\xi_1,\xi_2)$ are the unknown generalized displacements. In Eq. (1) the N_i are considered as parameters whose different values allow to build different order ESL shell theories. The shell theory corresponding to the expansion orders N_1 , N_2 and N_3 is denoted as $ED_{N_1N_2N_3}$.

To develop the proposed dG formulation, the shell reference domain Ω_ξ is partitioned into N_e elements Ω_ξ^e over which the generalized displacements $U_{ik}(\xi_1,\xi_2)$ are approximated via polynomial basis function.

Following Ref. [10] one obtains the following variational statement:

$$\begin{aligned}\sum_{e=1}^{N_e}\int_{\Omega_\xi^e}\left[\frac{\partial \boldsymbol{V}^T}{\partial \xi_\alpha}\left(\boldsymbol{Q}_{\alpha\beta}\frac{\partial \boldsymbol{U}_h}{\partial \xi_\beta}+\boldsymbol{R}_{\alpha 3}\boldsymbol{U}_h\right)+\boldsymbol{V}^T\left(\boldsymbol{R}_{3\alpha}\frac{\partial \boldsymbol{U}_h}{\partial \xi_\alpha}+\boldsymbol{S}_{33}\boldsymbol{U}_h\right)\right]d\,\Omega_\xi \\ +\sum_{i=1}^{N_i}\int_{\partial\Omega_{\xi I}^i}\mu[\![\boldsymbol{V}]\!]_\alpha^T[\![\boldsymbol{U}_h]\!]_\alpha d\partial\Omega_\xi+\sum_{e=1}^{N_e}\int_{\partial\Omega_{\xi D}^e}\boldsymbol{V}^T\boldsymbol{U}_h d\,\partial\Omega_\xi \\ =\sum_{e=1}^{N_e}\int_{\Omega_\xi^e}\boldsymbol{V}^T\overline{\boldsymbol{B}}d\,\Omega_\xi+\sum_{e=1}^{N_e}\int_{\partial\Omega_{\xi N}^e}\boldsymbol{V}^T\overline{\boldsymbol{T}}d\,\partial\Omega_\xi+\sum_{e=1}^{N_e}\int_{\Omega_{\xi D}^e}\mu\boldsymbol{V}^T\overline{\boldsymbol{U}}d\,\partial\Omega_\xi\end{aligned} \tag{2}$$

where the Einstein's summation is assumed for $\alpha,\beta = 1,2$, $\boldsymbol{U}_h$ is the vector containing the dG approximation of the generalized displacements field, $\boldsymbol{V}$ is the vector of the test functions chosen of the same form of the generalized displacements. In Eq (2), $\overline{\boldsymbol{B}},\overline{\boldsymbol{T}}$ and $\overline{\boldsymbol{U}}$ are the generalized domain forces, the generalized boundary forces and the boundary prescribed generalized displacements, respectively, $\boldsymbol{Q}_{\alpha\beta},\boldsymbol{R}_{\alpha 3},\boldsymbol{R}_{3\alpha}$ and $\boldsymbol{S}_{33}$ are the generalized stiffness matrices and $[\![\cdot]\!]$ is the interelement jump operator. Finally, $\partial\Omega_{\xi I}^i$, $\partial\Omega_{\xi D}^e$ *and* $\partial\Omega_{\xi N}^e$ are the inter-element interfaces and the elements portions of boundaries where the essential and natural boundary conditions are enforced, respectively. The definitions of the above-mentioned quantities can be found in Ref. [10], to which the reader is referred for the formulation details.

In the primal form of the proposed pure penalty dG method, namely Eq. (2), μ is the penalty parameter used to enforce the inter-elements continuity of the solution and the essential boundary conditions. The choice of μ is crucial for the method to be efficient as discussed in Refs. [7,10]

Applying the variational calculus procedures, the final nonlinear algebraic system is inferred, which is solved via a Newton-Raphson arc-length scheme. It is remarked that the use of a pure penalty formulation enables to compute the elements interface boundary integrals only once with the consequent computational time savings during the iterative solution scheme.

Numerical results

To illustrate the capabilities of the proposed method some results relative to the nonlinear analysis of cylindrical shells are proposed. The present results are representative of the proposed approach effectiveness that was proved by a lot of numerical tests whose results are not reported here for the sake of brevity.

Fig. 1a shows the geometry, boundary conditions and loads of the cylindrical shell having dimensions L = 254 mm, R = 2540 mm and θ = 0.1 rad and subjected to a central pinch load $\boldsymbol{F}$. Only a quarter of the structure is modelled for symmetry conditions. Three different shell sections have been considered: *i*) a single-layer section of isotropic material (see Table 1) and thickness τ = 6.35mm, which is labeled as C1 case (thin shell); ii) a single-layer section of isotropic material M1 (see Table 1) and thickness τ = 12.7mm, which is labeled as C2 case (moderately thick shell); iii) a three-layer section with [0/90/0] layup of 4.233 mm thick plies having orthotropic properties as M2 material in Table 1, which is labeled as C3 case.

Table 1: Material properties.

	Isotropic M1	Orthotropic M2
Young's modulus E_1	75000 [MPa]	3300 [MPa]
Young's moduli $E_2 = E_3$	75000 [MPa]	1100 [MPa]
Shear Moduli $G_{12} = G_{13} = G_{23}$	28846 [MPa]	660 [MPa]
Poisson's coefficients $\nu_{12} = \nu_{13} = \nu_{23}$	0.3	0.25

Fig. 1b show the transverse displacement of the load application point $u_3$versus the load amplitude F. These Equilibrium paths are computed using both the ED_{222} and ED_{333} shell theories and a 2 × 2 mesh grid of elements with polynomial trial and test function order p = 5. The results show that, as expected, snap-back or snap-through behavior occurs depending on the shell thickness ratio. The comparison of the present results with those of Refs [1], [11] and [12] shows very good agreement for both isotropic and multilayered shells, that proves the proposed approach effectiveness.

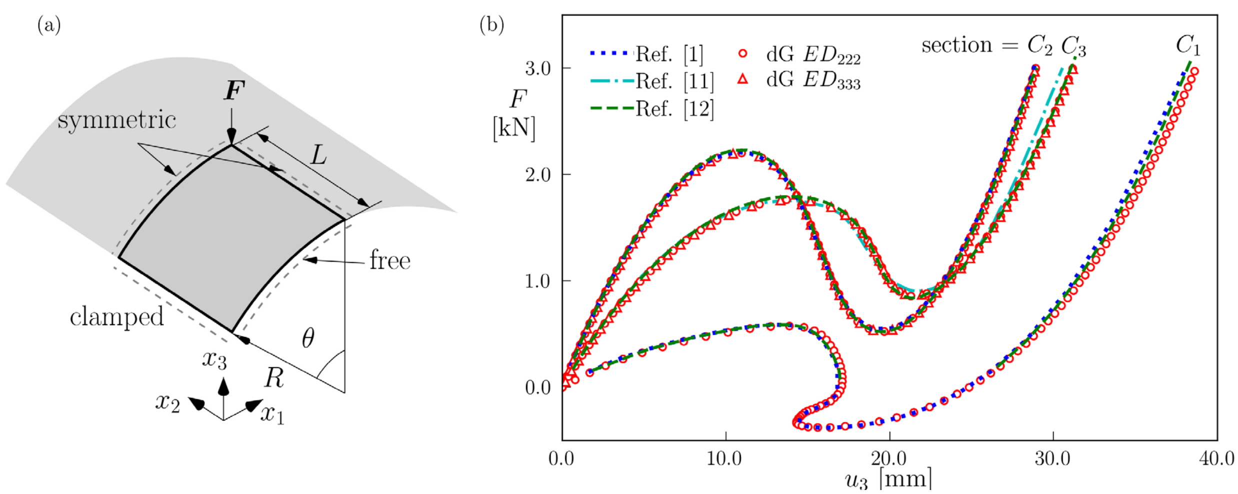

Figure 1: Cylindrical shells. (a) Geometry, boundary conditions and applied load. (b) Nonlinear equilibrium path for different shell sections.

Conclusions

A novel pure penalty discontinuous Galerkin method has been presented for the geometrically nonlinear static analysis of multilayered plates and shells described by refined equivalent single-layer kinematics with generality of the through-the-thickness resolution.

The numerical tests prove the ability of the method to deal with complex nonlinear behavior of shells and evidence very good agreement with literature results.

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