

Hybrid graph-clothoid based path planning for a fixed wing aircraft

Luciano Blasi^{1,a *}, Egidio D'Amato^{2,b}, Immacolata Notaro^{1,c}, Gennaro Raspaolo^{1,d}

¹Department of Engineering, University of Campania "L. Vanvitelli", Via Roma, 29, Aversa, 81031, Italy

²Department of Science and Technology, University of Naples "Parthenope", Centro Direzionale di Napoli, isola C4, Napoli, 80143, Italy

^aluciano.blasi@unicampania.it, ^begidio.damato@uniparthenope.it,

^cimmacolata.notaro@unicampania.it, ^dgennaro.raspaolo@studenti.unicampania.it

Keywords: Path Planning, Clothoids, RRT, UAVs

Abstract. Planning of safe and efficient trajectories is a critical task in the operation of unmanned aerial vehicles (UAVs), especially in urban or complex environments. With the increasing use of UAVs for various applications, such as surveillance, delivery, and inspection, it is becoming more important to automatically generate collision-free paths that also consider aircraft dynamics. This paper proposes an algorithmic approach based on the Rapidly exploring random tree (RRT) algorithm combined with a clothoid-based smoothing procedure to account for aircraft performance.

Introduction

The Rapidly exploring random tree (RRT) is a path planning algorithm commonly used in robotics and other applications where complex path planning is necessary. The algorithm constructs a tree of randomly generated exploration locations, iteratively adding nodes connected to existing ones if they do not cause collisions with their surrounding environment. RRT is particularly useful for trajectories planning in high-dimensional spaces or with complex obstacle spaces as it can quickly find approximate solutions to such problems. Typically, the solution produced by this algorithm is a piecewise linear path that overlooks the aircraft inability to follow instantaneous heading changes between segments. In literature, RRT is often hybridized with smoothing techniques based on Dubins circles or optimal control methods. The proposed method follows this idea, using clothoid curves to smooth the path. Clothoids, also known as Euler spirals, are mathematical curves that have a constant rate of curvature change along their length. They are widely used in various fields, including engineering, robotics, and transportation, due to their unique properties. Clothoids are particularly useful in path planning problems because they allow for smooth and continuous changes in curvature, enabling vehicles or robots to follow complex trajectories without abrupt changes in direction, which can be detrimental to performance and efficiency. The aim is to provide UAVs with the ability to navigate challenging environments while optimizing a flyable path that minimizes fuel consumption and avoids no-fly zones. In our approach, clothoid construction is integrated into an enhanced RRT procedure, with a reduction algorithm to avoid too many heading changes during flight.

Single aircraft clothoid-based path planning

To follow a sequence of waypoints, a flight trajectory can use straight and circular paths [1]. However, switching from one to the other is hard for fixed wing aircrafts because of the sudden change in yaw rate and bank angle. To track paths with continuous curvature and limits to maximum curvature and sharpness, clothoids can be used [2, 3]. These curves can achieve the desired direction with a linear relationship between curvature and arc length.

Spatial position coordinates x and y can be expressed as a function of the arc length s as follows [4]:

$$x(s) = x_0 + \int_0^s \cos\left(\frac{1}{2}\sigma\zeta^2 + \kappa_0\zeta + \psi_0\right) d\zeta \quad (1)$$

$$y(s) = y_0 + \int_0^s \sin\left(\frac{1}{2}\sigma\zeta^2 + \kappa_0\zeta + \psi_0\right) d\zeta \quad (2)$$

where σ is the sharpness, κ_0 represents the initial curvature, ψ_0 defines the initial heading and ζ is the integration variable. Multiple clothoids can be combined into a spline to create a continuous curvature path [5, 6, 7, 8] by matching the curvature at the tips of clothoid segments.

The maximum bank angle ϕ_{max} and the maximum bank angle rate $\dot{\phi}_{max}$ are important factors for setting the curvature and sharpness boundaries. The maximum path curvature depends on the aircraft speed v , the gravity acceleration g and the maximum bank angle ϕ_{max} [9]. The maximum sharpness can be obtained by taking the derivative of the curvature function with respect to time.

$$\kappa_{max} = \frac{g}{v^2} \tan(\phi_{max}) \quad (3)$$

$$\sigma_{max} = \frac{g}{v^2} \dot{\phi}_{max} \sec^2(\phi_{max}) \quad (4)$$

Being curvature a linear function with the arc length, the lowest and highest curvature values are found at the endpoints of a clothoid segment.

Rapidly exploring random tree (RRT)

In this chapter a RRT based path planning procedure is presented, combined with a reduction algorithm and clothoid smoothing curves to make the final paths more compliant with aircraft dynamics. RRT represents a wide spectrum of search algorithm extensively used in path planning [10]. Each RRT is based on a graph whose shape resembles a tree, built incrementally by adding random samples chosen from the search space. RRT is often build so that the graph tends to expand towards less explored areas; however, it is possible to properly guide the search towards specific areas of interest.

Let C be the search space, α an infinite sequence of samples in C and $\alpha(i)$ the i -th sample. The first vertex of the graph is q_0 and, starting from it, the tree is generated for k iterations. At every iteration a random sample is picked and linked to the nearest node of the graph, thus becoming a node itself.

To avoid too many changes of flight attitude a reduction algorithm is implemented inside the RRT. In a traditional RRT each new sample, out of the obstacle region, is linked only with the nearest node of the graph. With a reduction algorithm for each new point the most convenient path is searched; the starting point is the nearest node, then the search proceeds backwards by assigning a cost to each link between the new point and the nodes of the graph, provided the link does not enter an obstacle zone.

Only the link with the lesser cost is added to the graph and become one of its edges. During this phase the clothoids are inserted. Each edge between two nodes is composed by a linear path and two clothoids, one for each node. This is done to obtain a feasible trajectory without the need of a post processing phase.

Test case

The algorithm has been tested taking a real-world scenario. In particular, the city of Padova is considered and two of its squares, piazza Cavour and piazza degli Eremitani are chosen, respectively, as the starting point and the ending point, with a minimum height considered for the obstacles of 10 meters.

Since RRT is based on random samples, it is reasonable to expect some minor differences between different run of the algorithm, given that a solution (i.e. a path between the starting and ending points) is always found. A test was done by running the algorithm 100 times. The mean path length found was 456.62m while 200.24s was the mean computational time. On the other

hand, the minimum and maximum path length were, respectively, 417.52m and 606.34m, whereas the minimum and maximum time were, respectively, 44.04s and 806.32s. Figure 1 (Left) shows the effect of the reduction procedure. Without reduction the path (blue dashed line) would have been too long and convoluted, whereas the path obtained applying the reduction algorithm (red solid line) has just a few direction changes. In Figure 1 (Right) a detail of the smooth transition between different directions obtained with clothoid curves is shown.

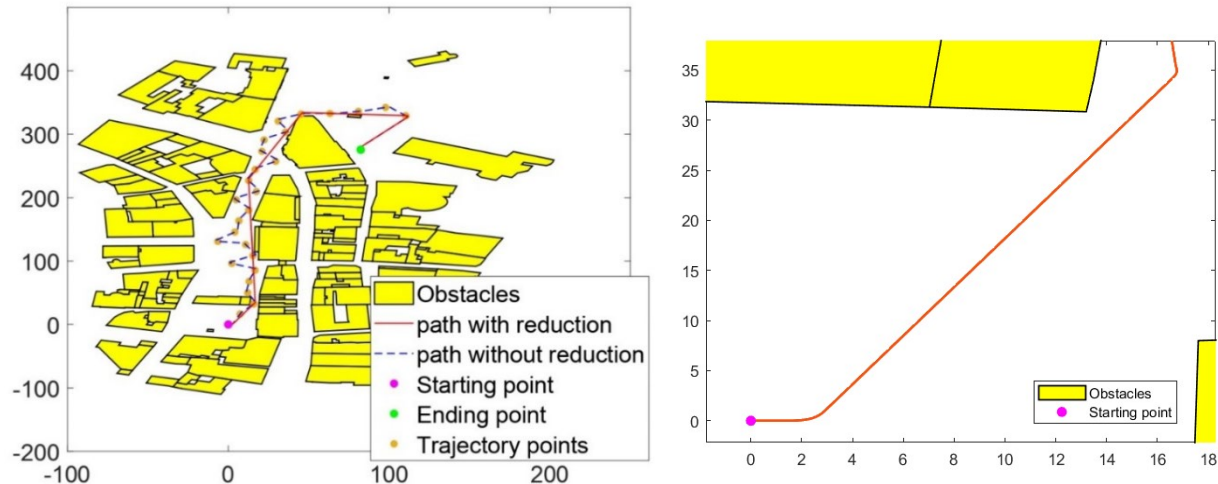


Figure 1 - (Left): comparison between paths obtained with (red solid line) and without (blue dashed line) the reduction algorithm; (Right): detail of the smooth transition between different directions obtained with clothoid curves.

Conclusions

This paper proposes an air vehicle path planner based on the Rapidly exploring random tree (RRT) algorithm combined with a clothoid-based smoothing procedure to better account for aircraft performance. The analysis carried out in the paper proved the effectiveness of the proposed procedure also in complex real-world scenarios, where the use of clothoids ensures the path compliance with aircraft dynamics. Moreover, the application of a reduction algorithm allows to avoid an excessive number of direction changes, although it increases the computational burden.

Future developments will be focused on the edge-adding procedure, being the basis of the algorithm, to guide the creation of new nodes more efficiently and minimize the length of the edges.

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