Materials Research Proceedings 37 (2023) 453-456 https://doi.org/10.21741/9781644902813-100

Refined structural theories for dynamic and fatigue analyses of structure subjected to random excitations

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Keywords: Random Excitation, Refined Structural Model

Abstract. This paper presents the application of low- and high-fidelity finite beam elements to analyze the dynamic response of aerospace structures subjected to random excitations. The refined structural models are developed with the Carrera Unified Formulation (CUF), enabling arbitrary finite element solutions to be easily generated. The solution scheme uses power spectral densities and the modal reduction strategy to reduce the computational burden. The response of an aluminum box beam is studied and compared with a solution obtained by a commercial code. Considering the root-mean-square value of the axial stress, an estimation of the fatigue life of the structure is obtained.

Introduction

Fatigue is one of the prevalent causes of failure in structural and mechanical components. To correctly estimate the fatigue life of an element, it is essential to evaluate the stress distribution as accurately as possible. Time and frequency domain analyses can be employed to characterize the fatigue performance of a structure. In particular, the frequency domain approach is preferred thanks to its lower computational cost than direct integrations of the governing equation in the time domain. [1-2]. In particular, the Power Spectral Density (PSD) method is commonly used in structural dynamics and random vibration analysis [3]. The Finite Element (FE) method is a flexible and powerful tool for determining displacement and stress spectra. Previous works mostly adopted finite elements based on classical and first-order shear deformation theories [4-5]. While these kinematics expansions are suitable for various structural problems, they may not hold valid assumptions for other applications, such as laminated and thin-walled structures. Two- and three-dimensional (3D) FE formulations can be employed to address their limitations, but they often lead to a significant increase in computational costs. This study proposes an alternative approach by utilizing high-order finite beam elements. These elements provide an accurate and computationally efficient solution for predicting structural responses to random excitations. The Carrera Unified Formulation (CUF) enables the automatic implementation of various kinematic models through a recursive notation. Specifically, the response of a clamped-free box beam is given in terms of PSD and root-mean-square (RMS) of displacements and stresses, and a fatigue life prediction is provided given the value of RMS of axial stress.

One-dimensional finite elements

The one-dimensional (1D) model adopted in this work is based on the Carrera Unified Formulation (CUF). According to the CUF, the 3D displacement field of a solid beam with main dimension along the y-axis, can be expressed as a generic expansion of the generalized displacements $u_\tau(y,t)$:

$$\mathbf{u}(\mathrm{x,y,z,t}) = \mathrm{F}_\tau(\mathrm{x,z})\mathrm{N}_\mathrm{i}(\mathrm{y})\mathbf{u}_\tau(\mathrm{y,t}), \qquad \tau = 1,2,\dots,\mathrm{M} \text{ and } \mathrm{i} = 1,\dots,\mathrm{nsn} \quad (1)$$

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In this equation, *M* represents the number of terms used in the expansion, while *nsn* represents the number of structural nodes of a single Finite Element (FE). Repeated subscripts indicate summation, $N_i(y)$ refers to the 1D FE shape functions, and $F_\tau(x, z)$ represents arbitrary functions on the cross-section. In this study, we adopt the Taylor (TE) and Lagrange (LE) expansion classes as polynomial bases. By introducing the Principle of Virtual Displacement (PVD) $\delta L_{int} = \delta L_{ext}$, it is possible to derive finite element matrices and vectors by assembling the so-called *Fundamental Nuclei*. These nuclei are the invariant of the methodology.

Theory of random response and fatigue life prediction

Using the Fourier transform of the equation of motion, it is possible to obtain the equation in frequency domain:

$$\mathbf{q_k}(\boldsymbol{\omega}) = [-\boldsymbol{\omega}^2\mathbf{M} + \mathbf{i}\boldsymbol{\omega}\mathbf{C} + \mathbf{K}]^{-1}\mathbf{F_k^*} \qquad \mathbf{i} = \sqrt{-1} \tag{3}$$

Where $\mathbf{q_k}$ is the column vector that collects the degrees of freedom (DOF) of the FE model, *k* is an arbitrary non-null generalized coordinate, $\mathbf{F_k^*}$ is the generalized force vector in frequency domain and it has only one nun-null term (equal to 1).

To reduce the computational cost, it is a common practice to employ a modal reduction strategy.

The Power Spectral Density (PSD) function of a signal gives an indication of the average power contained in particular frequencies and the root mean square (RMS) represents the square root of the area below PSD curve. Given the input PSD function of the load $\mathbf{S_F}$, the output PSD of the three-dimensional displacement $\mathbf{S_u}$ and the stress $\mathbf{S_\sigma}$ components at various frequencies ($\boldsymbol{\omega}$) are obtained:

$$\mathbf{S_{u_i}}(\boldsymbol{\omega}) = \overline{\mathbf{H}}_{\mathbf{u_i}}(\boldsymbol{\omega})\mathbf{S_F}(\boldsymbol{\omega})\mathbf{H_{u_i}^T}(\boldsymbol{\omega}) \qquad \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$$

$$\mathbf{S_{\sigma_j}}(\boldsymbol{\omega}) = \overline{\mathbf{H}}_{\boldsymbol{\sigma_j}}(\boldsymbol{\omega})\mathbf{S_F}(\boldsymbol{\omega})\mathbf{H_{\sigma_j}}^{\mathbf{T}}(\boldsymbol{\omega}) \qquad \mathbf{j} = \mathbf{1}, \dots, \mathbf{6} \tag{4}$$

where $\overline{\mathbf{H}}(\boldsymbol{\omega})$ and $\mathbf{H^T}(\boldsymbol{\omega})$ are the complex conjugate and the transpose of the transfer function and it can be computed with the FE method by performing as many frequencies response analysis as of the non-null terms (**nnz**) in the generalized force vector **F**:

$$\mathbf{H_{q_k}}(\boldsymbol{\omega}) = \left[\mathbf{q_{k_1}}\ \mathbf{q_{k_2}}\ \dots\ \mathbf{q_{k_L}}\right] \qquad \mathbf{k} = \mathbf{1}, \dots, \mathbf{nnz} \qquad \mathbf{L} = \mathbf{1}, \dots, \mathbf{fs} \tag{5}$$

where **q** is derived from Eq. (3) and *fs* is the number of frequency steps. In this work, the structure is subjected to white noise excitations, thus the PSD of this type of noise is constant.

Numerical results

The numerical example refers to a clamped-free box beam made of aluminum alloy with E = 71.7 GPa, G= 27.6 GPa, ν= 0.3, ρ = 2700 kgm^{-3} and dimensions L=2.00 m, h = 0.05 m, b = 0.25 m, t=0.01 m. The beam is subjected to three-point loads (1 N) as shown in Figure 1.

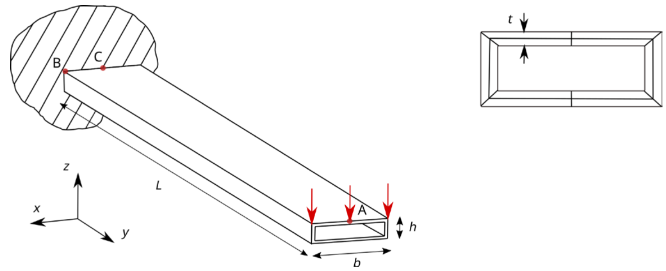


Figure 1. Boundary conditions and geometry of the clamped-free box beam subjected to clipped white noise. Scheme of the cross-section.

The structure was discretized using ten cubic beam elements. The first ten natural frequencies of the structure (Table 2) and the mass participation (Figure 2) were evaluated by conducting a normal mode analysis. The results have been obtained with the 12-LE9 model consisting of two Lagrange-type bi-quadratic elements for each lateral edge and four for the top and bottom surfaces. In the following analyses, forty modes were employed.

Table 1. First six natural frequencies obtained with NASTAN and CUF-FEM approach.

12-LE9	NASTRAN
13.90	13.88
56.13	56.17
84.03	83.09
178.68	170.8
223.17	216.08
324.78	324.67
406.34	378.13
461.23	424.19
613.37	539.52
629.52	563.01

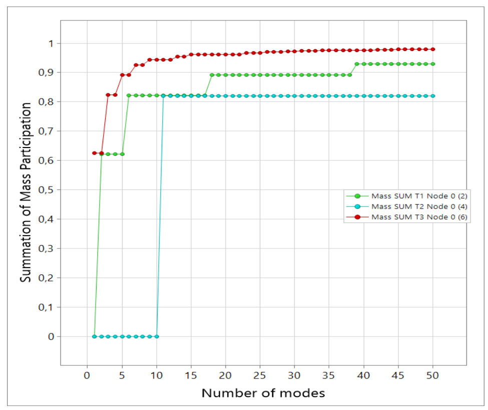
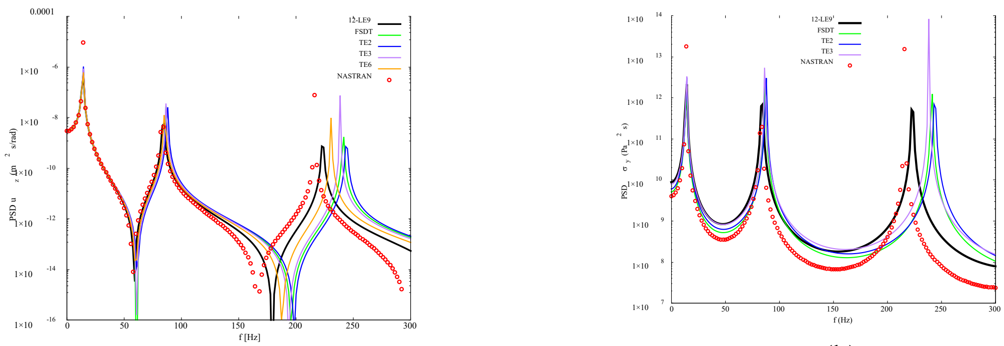


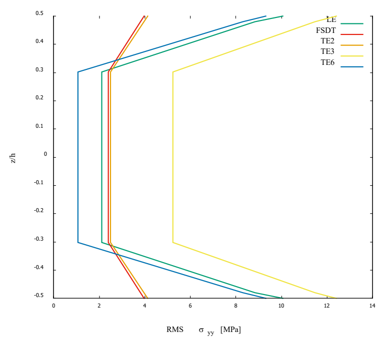
Figure 2. Mass participation versus number of modes of the box beam response.

Figure 3 shows point A's vertical displacement PSD and point C's axial stress PSD. In Figure 4, the distribution along the thickness corresponding to Point C of the root mean square of the axial stress is shown. Statistically speaking, the RMS stress value represents the 1σ value and will be experienced 68.3% of the time. A 2σ will be experienced 27.1% of the time, and a 3σ will be experienced 4.33%. These values represent 99.73% of the stresses the beam will experience at point C. Using Miner's cumulative damage $R_n = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3}$, it possible to obtained *n*, which is the number of cycles to fail: $1 = \frac{0.6831n}{N_1} + \frac{0.271n}{N_2} + \frac{0.0433n}{N_3}$. If the beam is vibrating at a frequency of 13.9 Hz (first natural frequency), then it will take approximately 2769 hours to fail. The results show that the 1D CUF approach is a valuable alternative to the common the 3D approach used by commercial codes.



(a) (b)

Figure 3. PSD of: (a) vertical displacement of point A, (b) axial stress of point B.



	1 RMS	2 RMS	3 RMS
S	10.06	20.12	30.18
N	4.3E10	1.8E8	7.6E6

Figure 4. RMS distributions of the axial stress along the thickness. On the right, a table with values taken from a fatigue curve of aluminium. For a given stress in [MPa], the number of cycles needed to cause failure is given.

Acknowledgments

This study was carried out within the MOST – Sustainable Mobility National Research Center and received funding from the European Union Next-Generation EU (PIANO NAZIONALE DI RIPRESA E RESILIENZA (PNRR) – MISSIONE 4 COMPONENTE 2, INVESTIMENTO 1.4 – D.D. 1033 17/06/2022, CN00000023). This manuscript reflects only the authors' views and opinions, neither the European Union nor the European Commission can be considered responsible for them.

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