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Coupled 3D peridynamics and refined 2D finite elements models embedded in a global\local approach

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Abstract. The present work proposes a two-step global\local approach for the three-dimensional analysis of structures . In particular, the first step makes use of classical finite element modelling, based on classical 2D elements, whereas a refined higher-order model based on the Carrera Unified Formulation (CUF) is coupled with a 3D peridynamics domain for the local analysis. The solution from the proposed method is compared with a full FEM solution. The objective of the present work is to pave the way to the building of a framework where coupled FE\PD models are embedded into this global\local approach to solve progressive failure problems.

Introduction

During the last century, various models have been proposed to describe the complex mechanism of the failure process. Most of them are based on the Finite Element Method (FEM), such as, for instance, the introduction of the Cohesive Zone Element (CZE) [1] and the eXtended Finite Element Method (XFEM) [2]. However, the main limitation of theories based on continuum mechanics is related to the difficulty in dealing with discontinuities, such as a crack. To overcome this obstacle, non-local theories have been widely used to solve fracture mechanics problems in the past years. For example, a recent non-local theory is Peridynamics (PD), introduced by Silling in [3]. According to PD, a solid body is made of physical particles, which interact in a pairwise manner when their distance is lower than a prescribed value. The main advantage of PD theory is the integro-differential nature of its governing equations, leading to the possibility of dealing with discontinuous displacement fields. Nevertheless, a characteristic of non-local theories is that they involve large, sparse, and not banded matrices, resulting in high computational demand.

In this context, researchers are working towards methodologies to couple FEM models based on classical elasticity with PD regions, intending to exploit the advantages of both methods. Pagani and Carrera [4] have proposed a coupling between the 3D peridynamics domain and 1D refined finite elements based on the Carrera Unified Formulation (CUF) [5]. This technique is based on Lagrange multipliers and has been successfully applied to progressive failure analysis [6].

This paper first proposes a coupling between 3D PD domains and 2D CUF-based higher-order elements. Then, this method is embedded in a two-step global\local approach [7,8]. The first step is a global analysis performed via commercial FEM software. Then, a region to be locally refined is chosen. After some manipulations, the displacements obtained from the global analysis are used as input for the local model, which is built by adopting CUF-based higher-order finite elements. This work wants to represent a first action towards creating a standalone framework, where coupled FE\PD models are embedded into this global\local approach to solve progressive failure problems.

Global\local approach

In the present research, a one-way global\local coupling technique is adopted. The procedure is divided into two steps. The first one consists of a global analysis of an entire structure through two-dimensional (2D) classical finite elements. The static analysis is performed through a commercial software based on Finite Element Method (FEM). Then, a region to be locally refined has to be chosen, according to an arbitrary criterion, such as for instance, the maximum stress. The second step of the global\local procedure consists in the creation of a refined local model, built using higher-order 2D plate finite elements, based on the Carrera Unified Formulation (CUF). In the CUF framework, the 3D displacement field $\boldsymbol{u}(x, y, z)$ is defined as a 1D through-the-thickness expansion function of the unknowns, evaluated through the finite element method. The relation can be expressed in the following way:

$$\boldsymbol{u}(x, y, z) = F_\tau(z) N_i(x, y) \boldsymbol{q_{\tau i}} \tag{1}$$

where F_τ is the expansion function, N_i the shape function and $\boldsymbol{q_{\tau i}}$ the nodal unknowns vector. The index τ indicates the number of terms in the thickness expansion, while the subscript i represents the number of the structural finite element nodes. In this work, sixteen-nodes cubic elements (Q16) are used as shape functions for the in-plane discretization, whereas four-node cubic Lagrange Expansion (LE3) are employed as expansion function F_τ over the thickness (see **Figure 1**).

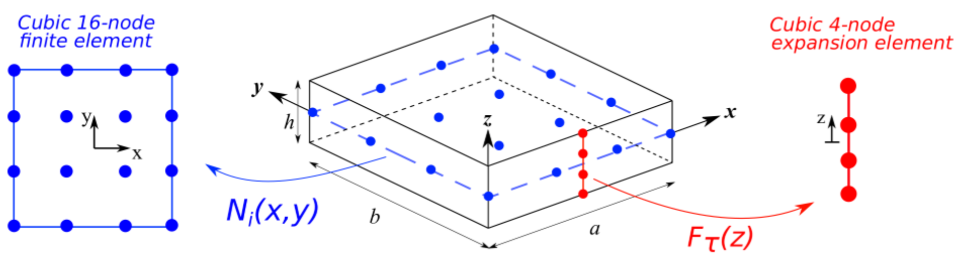

Figure 1. Graphical representation of the cubic interpolation for in-plane and thickness domain.

However, the Degrees of Freedom (DOFs) in the global model are expressed in terms of translational displacements (u_x^0, u_y^0, u_z^0) and rotations (θ_x^0, θ_y^0, θ_z^0) ,whereas the CUF-based local model makes use of a displacement-based formulation. Thus, a procedure to transform the rotations of the global model into pure displacement DOFs is conducted, by means of a Reissner-Mindlin displacement field. The equations read:

$$\begin{aligned} u_x(x, y, z) &= u_x^0(x, y) + z\theta_y^0(x, y) - y\theta_z^0(x, y) \\ u_y(x, y, z) &= u_y^0(x, y) - z\theta_x^0(x, y) - x\theta_y^0(x, y) \\ u_z(x, y, z) &= u_z^0(x, y) - x\theta_y^0(x, y) - y\theta_y^0(x, y) \end{aligned} \tag{2}$$

where quantities with superscript 0 denotes displacements and rotations of global DOFs. The computed pure displacement values from the global nodes are then interpolated and applied as boundary conditions into the refined local model, in order to perform a static analysis.

Coupling of 3D Peridynamics with 2D higher-order finite elements

Peridynamics is a non-local theory based on integro-differential equations. It assumes that a solid body is composed by material particles and each pair of those interacts if their distance is less than a material horizon of radius δ. The physical interaction between two particles at $\boldsymbol{x}$ and $\boldsymbol{x'}$ is called a *bond*, which extends over a finite distance. The non-local nature of peridynamics makes this theory attractive for being successfully applied to solid mechanics and crack propagation problems. Nevertheless, this method can be computationally prohibitive for many applications, because the resulting matrices are sparse, not generally banded, and large.

For this reasons, in previous works a coupling between CUF-based 1D finite elements and peridynamics has been proposed. In the present research, the extension to a coupling with 2D higher-order finite elements is proposed. This technique is based on the application of Lagrange multipliers at the interfaces $\mathscr{I}$ between 3D PD and FE domains (see **Figure 2**). In this way, the congruence conditions are satisfied, along with the elimination of the singularity in the global stiffness matrix. The linear system to be solved is expressed as follows:

$$\begin{bmatrix} \boldsymbol{K} & \boldsymbol{B}^{\boldsymbol{T}} \\ \boldsymbol{B} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix} \tag{3}$$

where $\boldsymbol{K}$ is the global stiffness matrix, including contributions from both FEM and PD domain, $\boldsymbol{B}$ is the coupling matrix and $\boldsymbol{\lambda}$ is the Lagrangian's force vector.

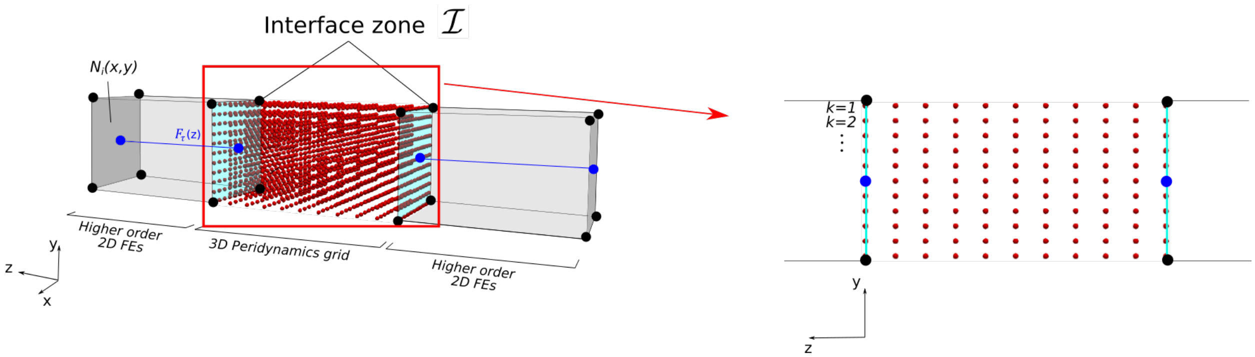

Figure 2. Coupling technique between higher-order 2D FEs and 3D peridynamics grid based on Lagrange multipliers.

Global\local approach and PD\FE coupled models framework

This research combines the G\L approach and the PD\FE coupled models within the same framework. The first step is still represented by the global analysis, performed via finite element analysis in commercial software using 2D classical finite elements. The main novelty lies in the nature of the local domain. In fact, the refined model consists of a 3D peridynamics grid coupled with a FE domain with 2D higher-order finite elements. In this work, a linear static analysis is performed on the local model by application of displacements and rotations from the global nodes as boundary conditions. However, this analysis can be seen as a first step for an iterative progressive failure algorithm, where the PD domain is used for modelling regions where cracks are likely to develop. The implementation of this framework has as objective the reduction of the computational costs of failure analysis while maintaining a high degree of accuracy.

Results

A representative case study is shown in this section. A plate subjected to localized transversal pressure is considered. It is made of an isotropic and homogeneous material, with Young Modulus $E = 200\ GPa$ and $\nu = 0.25$. The shape of the plate is rectangular, with sides $a = 200\ mm$ and $a = 300\ mm$. The thickness of the plate is equal to $h = 1.25\ mm$. A clamped condition is imposed in one edge, while a localized pressure equal to 10 kPa is applied on a corner element in the opposite edge. The mesh adopted for the global analysis is a 10×15 grid of 2D plate elements. A patch of nine elements is chosen as region to be locally analysed (see Figure 3). Finally, two different local domains have been investigated, one using only higher-order elements and another consisting in a PD\FE coupled model. A 5×5 grid with sixteen-nodes cubic element is adopted for the in-plane discretization, whereas a single four-node cubic element is used over the thickness. Concerning the PD domain, a grid spacing $\Delta x = 0.125\ mm$ and a horizon $\delta = 3\Delta x$ is chosen.

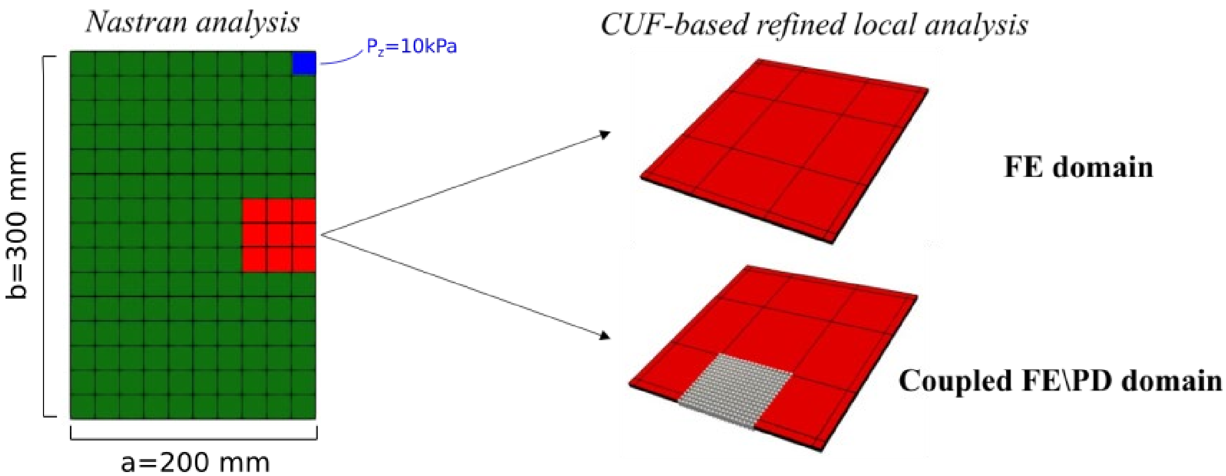

Figure 3. Geometry and modelling features for the global model and both FE and FE\PD local models.

Figure 4 depicts the displacement fields in the full FE and the coupled FE\PD model. A clear correspondence between the two cases is shown.

Moreover, in Figure 5 the distribution of vertical displacement along the blue line in the local domain is illustrated. The solutions from both cases are compared. The solid blue line represents the displacements in the full FE local domain, whereas the dots describe the transversal displacements in the coupled local domain. More specifically, the red dots are the displacement computed in the FE domain, whereas the green ones are those retrieved in the PD region. It is evident that the coupled FE\PD local analysis is able to accurately reproduce the behaviour of the full FE local domain.

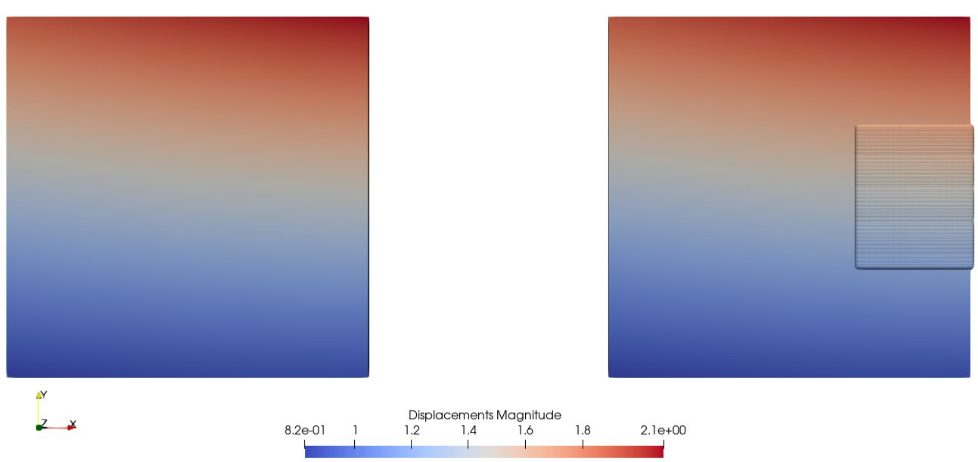

Figure 4. Displacement field in FE and coupled FE\PD local domains.

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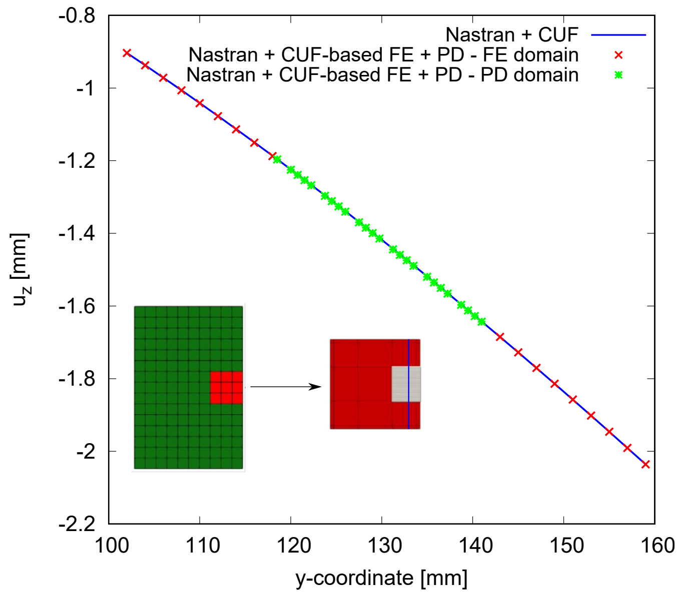

Figure 5. Distribution of transversal displacement along the blue line in the local domain. The solid blue curve represents the results from the full FE domain, whereas the red and green dots are the displacements computed in the coupled local region for FE and PD domains, respectively.

Conclusions

This manuscript has presented the pairing of a global\local approach and a model coupling Finite Elements (FEs) and peridynamics (PD). First, the coupling method between two-dimensional (2D) higher-order finite elements and a 3D peridynamics grid has been implemented for the first time. Then, a global\local analysis on an isotropic plate has been performed. The global analysis is performed via Nastran and by using 2D classical finite elements, whereas the refined local model is represented by a coupled FE\PD domain. Results show that this new coupled local model is able to accurately describe the displacement fields retrieved by a FE-based local analysis.

The presented technique represents a first fundamental step towards building a standalone framework, which will combine the global\local approach with FE\PD coupled models for progressive failure analysis, to reduce the computational costs while maintaining a high degree of solution accuracy.

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