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Best theory diagram using genetic algorithms for composite plates

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Abstract. Composite structures offer a practical approach for many engineering applications, but their design is complex and can result in excessive sizing due to limitations in current modeling techniques. BTDs minimize the number of unknown variables in a kinematic theory for desired accuracy or for a fixed error in the Carrera Unified Formulation. This paper presents a method for computing Best Theory Diagrams (BTDs) for laminated composite plates using Genetic Algorithms (GA). As reported in previous papers by the authors, a multi-objective optimization technique using a GA is applied to build BTDs for a given structural problem. The plate models stresses and displacements are compared to those of a reference solution, and a plate model performance is quantified in terms of the number of unknown variables, the mean error and standard deviation of the stresses and displacements. Also, with the objective of reducing the computational time, a Neural-Networks (NN) was trained to reproduce the mean error and standard deviation of the stresses and displacements for any plate model refined from a reference plate model is addressed. Numerical simulations were computed for laminated composite plates with previously uninvestigated boundary conditions and compare computational time for BTD calculation. The preliminary results show that the use of multi-objective GA plus NN method reduces considerably the computation time to build BTDs.

Introduction

To accurately predict stress distributions in modern composite structures, appropriate modeling formulations are necessary to account for their complex mechanical behavior, including both normal and transverse stress components. It is essential to have analysis tools that balance accuracy with computational efficiency. Consequently, the literature contains numerous articles on high-order models for composite structures [1]. The present work is embedded in the framework of the Carrera Unified Formulation (CUF). According to CUF, the displacement field for plate analysis is defined as an expansion of the thickness coordinate [2]. High-order theories are beneficial for response analysis, but they come at the cost of high computational expense. To address this issue, Carrera and Petrolo [3] developed the AXiomatic/Asymptotic method, which identifies unnecessary terms in a plate model for a specific output, such as displacement or stress. By eliminating these terms, a refined plate model with fewer unknown variables can be obtained without sacrificing accuracy. However, these refined plate models are problem-specific. Carrera and Miglioretti [4] extended the AXiomatic/Asymptotic method to create the Best Theory Diagram (BTD), which evaluates all combinations of terms in a full plate model. The BTD represents refined plate models in a plot that shows the number of terms versus the error. The "best" refined plate models with the least error for a fixed number of terms form the BTD.

Machine learning techniques such as neural networks and genetic algorithms have become increasingly popular in recent years for solving problems that involve a large number of variables, high levels of uncertainty, and rapidly changing behavior. These techniques have been applied in numerous fields, including computational mechanics, where they have been used to develop multiscale elements and data-driven solvers. In Ref. [4], a genetic algorithm was employed to construct Best Theory Diagrams (BTDs) with lower computational cost. Yarasca et al. [6] introduced a Multi-objective Optimization Method to create BTDs for sandwich plates, and later Mantari et al. [7] investigated the use of neural networks to reduce computation time in BTD construction.

In this study, a method for computing Best Theory Diagrams (BTDs) for composite plates using Genetic Algorithms (GA) is presented. This study uses mathematical formulation and benchmarks from prior research [7], with a new GA technique [8] and cost function for optimization. The study will perform numerical simulations for laminated composite plates with previously uninvestigated boundary conditions and compare computational time for BTD calculation. Also, the use of Neural Network models combined with Genetic algorithms is investigated.

Preliminaries

The Best Theory Diagrams (BTDs):

The construction of reduced models through axiomatic/asymptotic methods, can lead to a diagram in which, for a given problem, each reduced model is associated with the number of active terms and its error computed on a reference solution. This diagram, Fig.1, allows editing an arbitrary given theory to get a lower number of terms for a given error, or to increase the accuracy while keeping the computational cost constant. Considering all the reduced models, it is possible to recognize that some of them provide the lowest error for a given number of terms. These models represent a Pareto front for this specific problem. As in Ref. [4], the Pareto front is defined as the best theory diagram (BTD). This curve is case-dependent since it changes for several problems, i.e., different materials, geometries, boundary conditions, and output parameters. If a single output parameter is selected (only one displacement or one component of the stress tensor), the BTD considers the number of active terms and the error of the selected output parameter computed on a reference solution. Investigations of this sort have been reported in Ref. [9]. It is worth noticing that, although the output parameter may be freely selected, only one at a time can be investigated. This latter limitation has been removed by the multi-objective BTDs proposed by Mantari et al. [10], where multiple output parameters can be investigated in a single analysis.

Methodology and Proposed Algorithm

Mathematical Model: Carrera Unified Formulation for plates:

Figure 2 illustrates the geometry and coordinate system of a multilayered plate comprising L layers, where the in-plane coordinates are denoted by x and y, and the thickness coordinate is denoted by z. The layer number, denoted by the integer k, represents the layer's position in the multilayered plate, counting from the bottom to the top surface.

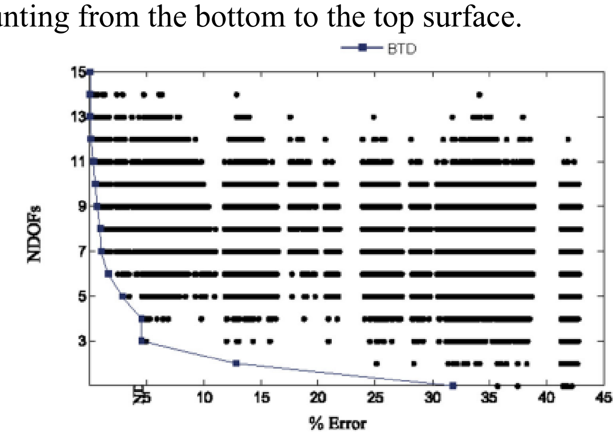

Figure 1: Body Theory Diagram on ED4, aluminum path, a/h=2.5, Mantari et al. [10].

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According to CUF, the displacement field of a plate structure can be written as follows:

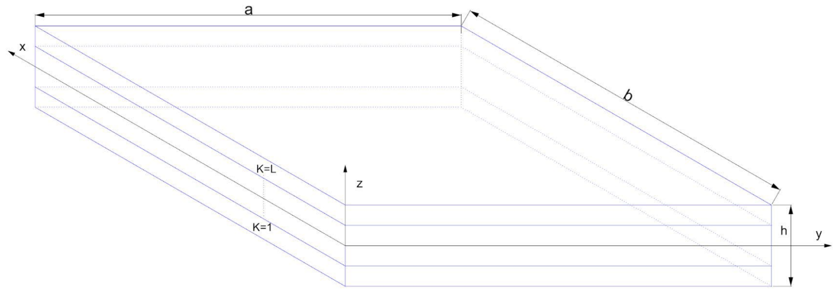

Fig. 2. Plate geometry and reference system.

$$\begin{cases} u_x(x,y,z) = F_1(z)u_{x_1}(x,y) + F_2(z)u_{x_2}(x,y) + \cdots + F_{N_{\text{exp}}}(z)u_{x_{N_{\text{eqp}}}}(x,y), \\ u_y(x,y,z) = F_1(z)u_{y_1}(x,y) + F_2(z)u_{y_2}(x,y) + \cdots + F_{N_{exp}}(z)u_{y_{N_{eq}}}(x,y) \\ u_z(x,y,z) = F_1(z)u_{z_1}(x,y) + F_2(z)u_{z_2}(x,y) + \cdots + F_{N_{\text{exp}}}(z)u_{z_{\text{exp}}}(x,y) \end{cases} \tag{1}$$

In compact form:

$$\boldsymbol{u}(x,y,z) = F_\tau(z) \cdot \boldsymbol{u}_\tau(x,y)\tau = 1, z, \dots, N_{\text{exp}} \tag{2}$$

where $\boldsymbol{u}$ is the displacement vector whose components (u_{x}; u_{y}; u_{z}) are the displacements along the x, y and z reference axes. F_τ are the expansion functions and $\boldsymbol{u}_\tau$ (u_{x_τ} ; u_{y_τ} ;u_{z_τ}) are the displacements variables. N_{exp} is the number of terms of the expansion. According to the equivalent single layer scheme, a multilayered heterogeneous plate is analyzed as a single equivalent lamina. In this case, F_τ functions can be considered as functions of z defined as $F_\tau = z^{\tau-1}$. The number of unknown variables is independent of the number of plate layers. The equivalent single layer models are indicated as EDN, where N is the expansion order. In this paper, the 'zig-zag' function proposed is employed. The equivalent single layer models considering Murakami 'zig-zag' function are indicated as EDZN. An example of an EDZ4 displacement field is reported as:

$$\begin{aligned} u_x &= u_{x_1} + zu_{x_2} + z^2u_{x_3} + z^3u_{x_4} + (-1)^k\zeta_k u_{x_5} \\ u_y &= u_{y_1} + zu_{y_2} + z^2u_{y_3} + z^3u_{y_4} + (-1)^k\zeta_k u_{y_5} \\ u_z &= u_{z_1} + zu_{z_2} + z^2u_{z_3} + z^3u_{z_4} + (-1)^k\zeta_k u_{z_5} \end{aligned} \tag{3}$$

where $\zeta_k = 2z_k/h_k$ is a non-dimensional layer coordinate and h_k the thickness of the k-layer. On the other hand, layer-wise models can be conveniently built using Legendre's polynomials expansions in each layer. Detailed description of equation derivation can be found in [7].

In this paper, layer-wise models are denoted by the acronym as LDN, where N is the expansion order. An example of LD4 layer displacement field:

$$\begin{aligned} u_x^k &= F_t u_{x_t}^k + F_2 u_{x_2}^k + F_3 u_{x_3}^k + F_4 u_{x_4}^k + F_b u_{x_b}^k \\ u_y^k &= F_t u_{y_t}^k + F_2 u_{y_2}^k + F_3 u_{y_3}^k + F_4 u_{y_4}^k + F_b u_{y_b}^k \\ u_z^k &= F_t u_{z_t}^k + F_2 u_{z_2}^k + F_3 u_{z_3}^k + F_4 u_{z_4}^k + F_b u_{z_b}^k \end{aligned} \tag{4}$$

Finite element approximation:

A classical Finite Element technique is used to easily deal with arbitrary shaped cross-sections. The generalized displacement vector is given by:

$$\boldsymbol{u}_\tau(y) = N_i(y)\boldsymbol{q}_{\tau i} \tag{5}$$

where N_i are the shape functions and $q_{\tau i}$ is the nodal displacement vector:

$$\boldsymbol{q}_{\tau i} = \left\{ q_{u_{x_{\tau i}}} q_{u_{y_{\tau i}}} q_{u_{z_{\tau i}}} \right\}^T \tag{6}$$

For the sake of brevity, the functions are not listed here, they can be found in Carrera Formulation. The functions are defined in the natural coordinates and transpose in the real coordinate in according with the isoperimetric formulation. The stiffness matrix of the elements and the external loadings are obtained via the Principle of Virtual Displacements (PVD):

$$\delta L_{int} = \int_V \left(\delta \boldsymbol{\varepsilon}_p^T \boldsymbol{\sigma}_p + \delta \boldsymbol{\varepsilon}_n^T \boldsymbol{\sigma}_n \right) dV = \delta L_{ext} \tag{7}$$

Where L_{int} stands for the strain energy, L_{ext} is the work of external loadings and δ stands for virtual variation. The PVD for a multilayered plate structure reads, a detailed derivation of equation can be found n [7]:

$$\delta \boldsymbol{q}^{ksj}: \qquad \boldsymbol{K}^{k\tau sij} \boldsymbol{q}^{k\tau i} = \boldsymbol{P}^{ksj} \tag{8}$$

where $\boldsymbol{P}^{ksj}$ is a 3 x 1 matrix, called fundamental nucleus of the external load. $\boldsymbol{q}^{k\tau i}$ and $\delta \boldsymbol{q}^{ksj}$ are the nodal displacements and its variation respectively.

Proposed Genetic Algorithm Optimization Method:

Refined plate theories provide improved accuracy and the ability to detect non-classical effects, but the higher number of displacement variables required leads to higher computational costs. To minimize the computational cost required to construct a Best Theory Diagram (BTD), a genetic algorithm (GA) is employed. The GA evaluates a set of random refined models, referred to as the population, over multiple generations until the final generation's BTD converges. The Axiomatic/Asymptotic method, developed by Carrera and Petrolo [3], addresses this issue by enabling the identification of the role of each variable in computing a specific displacement stress output variable. This method involves evaluating every potential plate model combination resulting from deactivating each term. Hence, the number of evaluations needed is potential, 2^M , where *M* the number of deactivated terms. A graphical notation is introduced to represent the results. This consists of a table with three rows, and some columns equal to the number of the displacement variable used in the expansion. As an example, an LD4 model for a two-layer plate is shown in Table 1 (full model). Table 1 also shows a refined model in which the term in the first, where, squared symbols (■) means Non-deactivable term (due to material continuity), empty-triangle (△) is Inactive term, and filled-triangle (▲) is active term.

Table 1: Example model representation.

Full model representation									Reduced model representation								
■	▲	▲	▲	■	▲	▲	▲	■	■	▲	▲	▲	■	△	▲	▲	■
■	▲	▲	▲	■	▲	▲	▲	■	■	▲	▲	▲	■	▲	▲	▲	■
■	▲	▲	▲	■	▲	▲	▲	■	■	△	▲	▲	■	▲	▲	▲	■

For multiple displacement/stress outputs, each output parameter has a given error which is computed simultaneously. The optimization method objectives functions are the number of terms in the refined models, the mean error and the standard deviation of the stresses and displacements. After the Axiomatic/Asymptotic method is employed, the mean error and the standard deviation is replaced by a new objective function denoted by $\%Error_{ul}$. $\%Error_{ul}$ is the sum of the mean error and the standard deviation. In this study the error is defined as:

$$\% \, Error = 100 W_r \frac{\sum_{i=1}^{N_p} \left| Q^i - Q_{ref}^i \right|}{max\left| Q_{ref} \right| \cdot N_p} \quad (9)$$

where W_r is the vector of weights for the new optimization function, as has the value of:

$$W_r = \begin{cases} 2, & if\ r = 1,2,3 \\ 1, & otherwise \end{cases}$$

This weight gives priority to the u_x, u_y and u_z error: The used of weighs to improve estimations was proposed in [11].

In this way, a 2-dimensional Pareto front, the so-called BTD, is built selecting the best plate theories considering the objective functions: number of terms and $\%Error_{ul}$. It is important to remark that $\%Error_{ul}$ is not exactly the upper limit error of the output parameters, but an indicator employed for comparative purposes.

In this study, the Kerry-Lothrop GA from [8] is utilized due to its demonstrated high performance across various research areas. Typically, the classical approach involves converting all variable types to binary design variables, either explicitly or through a user interface. This conversion process was implemented using the Matlab Genetic Algorithm Optimization Toolbox. The Multi-objective optimization technique flowchart is presented in Fig. 3. In this procedure, the most time-consuming step is the FEA of the refined plate models. If the number of generations and/or the population size is too large, the procedure can become unviable. For this reason, a Neural-Network (NN) is implemented to replace the FEA, as was addressed in Yarasca et al. [7]. The NN consists of simple processing units, the neurons, and directed, weighted connections between those neurons. The neurons are connected to different layers. Once the number of layers and neurons per layer are set, the NN starts the training procedure. The training is an iterative process. To initiate the training of a neural network (NN), a set of initial weights is randomly selected in the first iteration. A fixed number of random cases with predetermined inputs and outputs are used to compute the weights. Next, the NN output for the given inputs is compared to the desired outputs, and the errors are then propagated back to the NN to adjust the weights. This process continues for each iteration until the NN reaches an acceptable global error and converges. In the current study, a population of 1000 and 100 generations were used for the genetic algorithm (GA), and the NN utilized 2000 training samples. The NN architecture consisted of 15 neurons with three layers for EDZ4 models and four layers for LD4 models.

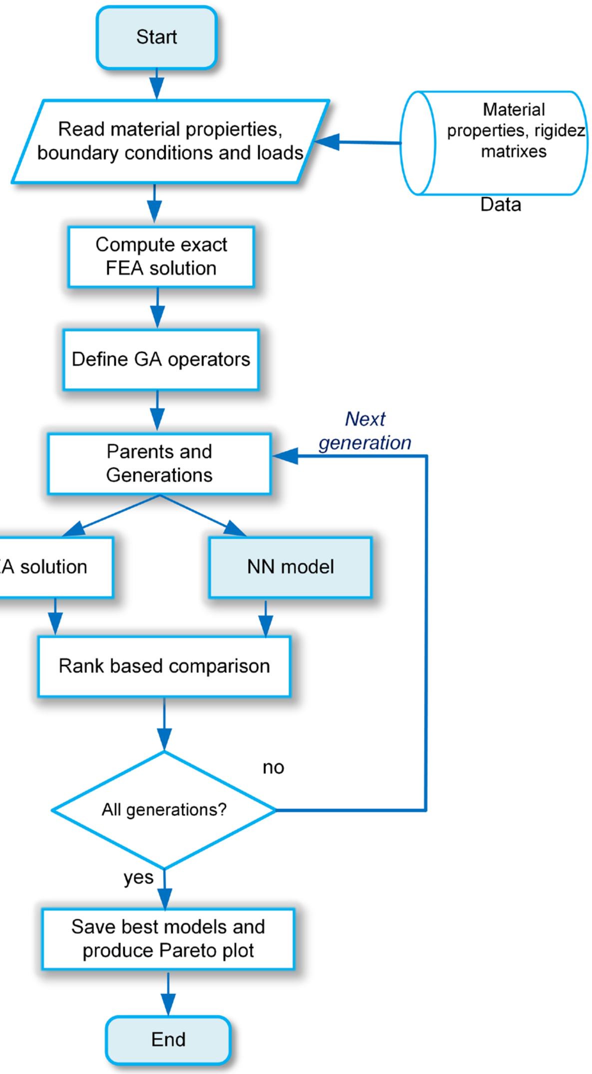

Figure 3: Multi-objective optimization technique flowchart, using FEA and Neural Networks

Numerical Results and Discussion:

This section presents the results obtained from the new multi-objective optimization technique. A transverse uniformly distributed pressure was applied at the top surface of a square composite plate with equal side lengths *(a = b)* and thickness (*h*), as shown in Figure 2. The study focused on a laminated composite plate subjected to three different sets of boundary conditions: simple support on all four sides, clamped on all four sides, and clamped-free with opposing sides having the same boundary condition (CFCF). Due to space limitations, this paper only presents the results for CFCF, while the complete set of results will be presented in a forthcoming publication.The reduced models are developed for the displacements u_x, u_y, u_z, and the stresses $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{xy}, \tau_{xz}$ and τ_{yz}. The following normalized quantities are defined for the displacements and stresses:

$$\bar{u}_x = \frac{u_x \cdot E_2^{k=1} \cdot h^2}{\bar{p}_z \cdot a^3}, \bar{u}_y = \frac{u_y \cdot E_2^{k=1} \cdot h^2}{\bar{p}_z \cdot a^3}, \bar{u}_z = \frac{u_z \cdot 100 \cdot E_2^{k=1} \cdot h^3}{\bar{p}_z \cdot a^4} \tag{10}$$

$$\bar{\sigma}_{xx,yy} = \frac{\sigma_{xx,yy}}{\bar{p}_z \cdot (a/h)^2}, \bar{\sigma}_{zz} = \frac{\sigma_{zz}}{\bar{p}_z}, \bar{\tau}_{xy} = \frac{\tau_{xy}}{\bar{p}_z \cdot (a/h)^2}, \bar{\tau}_{xz,yz} = \frac{\tau_{xz,yz}}{\bar{p}_z \cdot (a/h)},$$

where k =*1* identifies the bottom layer; $\bar{u}_x$ and $\bar{\tau}_{xz}$ are calculated at *x=0*, *y=b/2*; $\bar{u}_y$ and $\bar{\tau}_{yz}$ are calculated at *x=a/2, y=0*; $\bar{u}_z, \bar{\sigma}_{xx}, \bar{\sigma}_{yy}$ and $\bar{\sigma}_{zz}$ are calculated at *x= a/2, y=b/2* and $\bar{\tau}_{xy}$ is calculated at *x=y=0*. The stresses $\bar{\tau}_{xz}$, $\bar{\tau}_{yz}$ and $\bar{\sigma}_{zz}$ obtained from the EDZ4 and LD4 plate model were computed via the indefinite equilibrium equations of 3D elasticity. In this paper, an LD4 model was employed as the reference solution. Also, comparisons with the work presented at Mantari et al. [7] are addressed. The results reported in Refs. [12] show that the LD4 plate model is in good agreement with the three-dimensional exact elasticity result. The GA used here was designed by Lothrop (2003) and suffered only minor changes in order to be applied to this problem. The floating-point representation was chosen for all variables. Those parameters that were problem specific are provided in Table 2.

Table 2: Considered genetic algorithms parameters.

Population	1000
Generations	30
Xover	70% for 30 pairs
Mutate	5%

Laminated Plates:
Laminated composited plates with different length-to-thickness ratios, boundary conditions, and lamination sequences were investigated using an EDZ4 plate model. In the examples considered, the individual laminae were considered of equal thickness and the following set of material properties was used for each lamina: $E_{L/E_T} = 25; G_{LT}/E_T = 0.5;\ G_{TT}/E_T = 0.2; \nu_{LT} = \nu_{TT} = 0.25$. The length-to-thickness ratio *a/h=5* and lamination sequences 0/90, and the combinations of boundary conditions *CFCF*, were studied. The BTDs for the case studies are presented in Fig. 4. From this plot as was expected, GA+NN is faster but has the drawback of being a little more errors in the models optimized. The refined plate model's accuracies are reported in Table 3. Selected displacements and stress through the thickness distributions are shown in Figs. 5. The notation used is the following: the refined models built are indicated as N-*hybrid* refined model (N-HRM); where *N* is the number of variables in the HRM. The reference solution (in this case, LD4) is included for comparison purposes. To verify the correct convergence of the GA and NN to the Axiomatic/Asymptotic method's BTD, a comparison between the different BTDs is shown in Fig. 4. The BTD denoted by AAM (Axiomatic/ Asymptotic method) is built evaluating every refined model from the full EDZ4 plate model. As can be observed, the BTDs obtained are in complete agreement. Fig. 4 shows that the NN precision improves for high length-to-thickness and simply supported boundary conditions.

For the sake of reproducibility, the selected refined plate models from the BDTs are reported in Tables 3. The number of active terms is indicated by M_E. This is expected since for high length-to-thickness ratio and simply supported boundary conditions, the number of refined plate models with adequate results in the population increases. Therefore, the NN prediction improves because of the reduction in population performance variability. For that reason, hereafter only low length-to-thickness ratios are investigated.

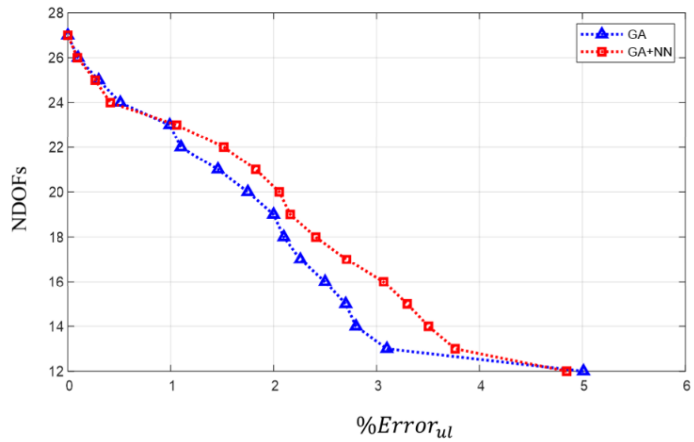

Figure 4. BTDs for laminated composite plate, 0/90, a/ h=5, CFCF.

Table 3: Refined EDZ4 models, laminated composite plate, 0/90, a/h=5, CFCF.

M_E=15/27	M_E=21/27
■ ▲ ▲ △ ■ △ ▲ △ ■ ■ ▲ △ △ ■ ▲ △ △ ■ ■ ▲ △ △ ■ △ △ △ ■	■ ▲ ▲ ▲ ■ △ ▲ ▲ ■ ■ ▲ △ ▲ ■ ▲ △ △ ■ ■ ▲ ▲ △ ■ △ ▲ ▲ ■
M_E=24/27	M_E=26/27
■ ▲ ▲ ▲ ■ ▲ △ ▲ ■ ■ ▲ ▲ △ ■ ▲ ▲ △ ■ ■ ▲ ▲ ▲ ■ ▲ ▲ ▲ ■	■ ▲ ▲ ▲ ■ ▲ ▲ ▲ ■ ■ ▲ ▲ ▲ ■ ▲ ▲ △ ■ ■ ▲ ▲ ▲ ■ ▲ ▲ ▲ ■

The NN can predict the results in terms of mean error and standard deviation with acceptable accuracy. Concerning the EDZ4 plate model, Fig. 5 and Table 4 show that EDZ4 refined plate models are insufficient to simulate laminated plates with CFCF boundary conditions and asymmetric laminations such as 0/90. The NN implementation makes the multi-objective optimization method practical in terms of computational cost. In this study, the computation cost for the GA population presented in Table 2 and FEA solution are presented in Table 5. In this table for $M_E = 5$ is 16.1 minutes and $M_E = 15$ is 23.3 minutes. The computational time is reduced when NN, The NN can predict the results in terms of mean error and standard deviation with acceptable accuracy, the Neural Network herein used in a NN trained to reproduce the mean error and standard deviation of the stresses and displacements, the mode is composed by set to 15 neurons with 3 layer. The new computation time are for $M_E = 5$ is 8.2 minutes and $M_E = 15$ is 9.6 minutes. Table 5 also shows a comparison of the computational cost between the present work and the method presented in Mantari et al. [7]. From this table, it is possible to observe a better performance of the presented work. However, it is important to notice that this method computes one theory for a determined number of active terms, while the Mantari et al. [7] method performs all the theories at once.

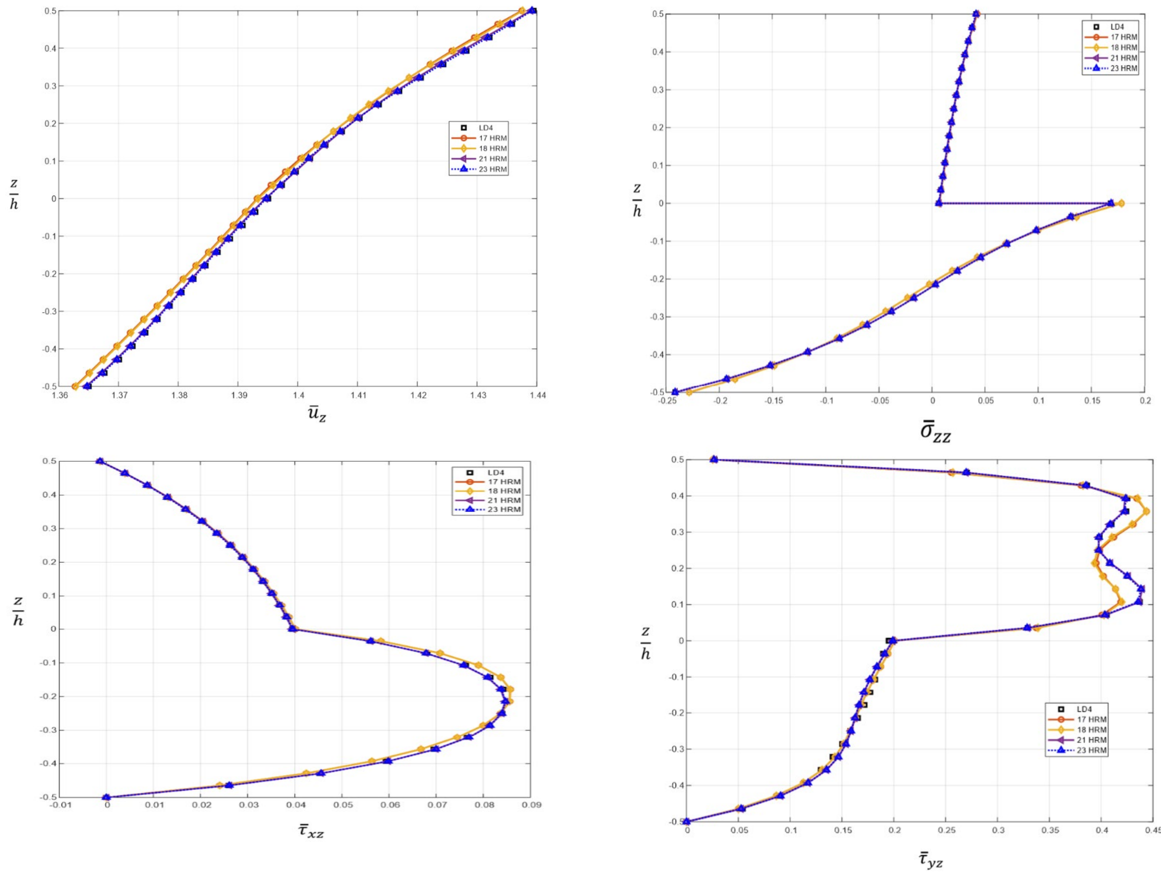

Figure 5: Selected displacement and stresses distribution for laminated composite plate, 0/90, a/h=5,CFCF.

Table 4: Error percentages of the refined EDZ4 models, laminated plate, 0/90, a/ h=5, CFCF.

a/h=5									
CFCF									
M_E	$\bar{u}_x$	$\bar{u}_y$	$\bar{u}_z$	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$	$\bar{\sigma}_{zz}$
5	0.853	1.299	0.717	1.031	1.619	1.752	1.164	3.785	3.007
7	0.741	0.062	0.128	1.064	0.138	1.688	1.221	1.641	2.401
10	0.360	0.013	0.033	0.157	0.024	1.009	0.217	0.489	1.628
12	0.176	0.009	0.021	0.101	0.011	0.348	0.158	0.488	0.499
15	0.135	0.000	0.000	0.002	0.000	0.226	0.002	0.001	0.003

Table 5. Computation time for laminated composite plate, 0/90, a/ h=5, CFCF

Number of active terms	GA+FEA	GA+NN
M_E=5	16.1 min	8.2 min
M_E=15	23.3 min	9.6 min
Mantari et al. [7], raw	90 min	6.71 min
Mantari et al. [7], optimized	90 min	4.42 min

Conclusions and Future work

In this article, we describe the development of equivalent single layer and layer-wise plate models for laminated composite plates, which offer 3D-like accuracy and optimized computational cost. The Best Theory was built using the axiomatic/asymptotic method, a genetic algorithm, and the neural network, with output parameters including all displacements and stresses. By replacing finite element analysis with a neural network, computational time is drastically reduced. Our results present the BTDs, displacements and stresses for a 0/90 Laminated composite plate CFCF boundary condition. The main conclusion drawn from our findings is that the implementation of the neural network significantly reduces the computational time required to build BTDs. Also, this study proposes a different GA algorithm [8] with new optimization function, using weights, than in Mantari [7], and better performance in computational cost was found. Future work will be concentrated is carry out numerical simulations for more benchmarks, different boundary conditions and different materials.

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