# Investigation of electromechanical coupling characteristics of a double magnet system

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**Abstract.** In this work, the experimental results of the electromechanical coupling coefficient identification are presented. The research covers two cases: test with a single magnet (I) and test with a double magnet, where two repelling magnets in one structure are connected (II). In case (I), the analytical description of the electromechanical coupling coefficient was determined. Whereas the analysis of case (II) confirms the superposition principle for interactions between both magnets and a single inductive coil. Finally, the presented analysis proposes some premises which will be used in the future to develop the model with two levitating magnets.

## Introduction

Energy harvesting from a system with one levitating magnet is well-known and has been described in many papers [1, 2, 3]. This research presents many aspects: the modelling of electromechanical coupling [1], the existence of electrical damping [2], optimal resistance load [3], etc. However, one of the problems of this solution is its low recovery effectiveness. For example, a small electromagnetic harvester dedicated to energy recovery from human body motion is presented in [4]. Obtained results showed that the maximum harvested electrical power was approx. 0.7mW. In order to increase efficiency, modifications to the electromagnetic harvester structure are introduced. One of the suggestions is to use two independent levitating magnets. This concept is presented by Mitura and Kecik in [5]. This research has several limitations. First, the electromechanical coupling coefficient has a constant value, because small vibrations from magnets are only considered. Secondly, each magnet has its inductive coil and the determination of the total electrical power is ambiguous. However, the results obtained are promising. The maximum recovered electrical power was about 0.45W. In the next research, it is planned to use only one coil to induce an electromotive force from the larger vibrations of both movable magnets. Developing a new model requires describing the strongly nonlinear curve of the electromechanical coupling coefficient. Moreover, the mechanism of the electromotive force generation should be checked, i.e. is it a superposition of the interactions between individual magnets and a single coil.

The research shown in this paper will be used in the future to develop a model with two levitating magnets and a single inductive coil. The presented analysis is limited to two selected issues from the initial modelling stage (sections 2 and 3). In the section "Experimental research with a single magnet" the experimental measurement of the electromechanical coupling coefficient for the motion of a single magnet relative to the coil is presented. The obtained nonlinear relationship is approximated by analytic functions. This description can be used to model the electromechanical coupling coefficient of interaction between one magnet and a coil. The next section "Experimental research with double magnet" presents experimental research, where two moving magnets were used. The purpose of this study was to show that descriptions of the interaction between each magnet and the coil can be considered separately. Tests for two independent magnets would be inconclusive. Therefore, the special case was analyzed. In this case, both magnets were connected by a connector. Finally, the distance between both magnets is constant. The created structure (magnet-connector-magnet) called a double magnet is easier to analyze, and the superposition principle could be confirmed. The superposition principle means that the coil voltage obtained from the double magnet motion results from the sum of the voltages generated by two single magnets considered separately. In the last section, the conclusions are presented.

## Experimental research with a single magnet

All experimental research was performed on a small strength machine, Shimadzu. An epoxy tube was clamped in the bottom fixed machine handle. Next, an inductive coil was installed on this tube. The basic parameters of the applied ring-shaped coil were: total length - 50 mm, inner diameter - 28 mm, outer diameter - 42 mm, resistance - 1.15 kΩ and inductance - 1.46 H. The tube also provided proper guidance of the neodymium magnet relative to the coil. The cylindrical magnet used was 20 mm in diameter and 20 mm in height and was connected to the machine traverse. During tests, this traverse moved up and down at a constant speed (1 meter per 1 minute) and generates the motion of the magnet relative to the coil. Based on the experiment, the electromotive force $U$ (coil voltage in volts) and magnet position relative to the coil centre $x$ (distance in millimetres) were measured. Obtained nonlinear curve $U=f(x)$ is presented in Fig.1a. These data were used for the determination of electromechanical coupling coefficient $\alpha$. Simple calculations can be made from the following equation:

$$\alpha=U/v, \tag{1}$$

where $v$ is the traverse speed (0.0167 m/s). In Fig. 1b the nonlinear characteristics $\alpha=f(x)$ can be seen.

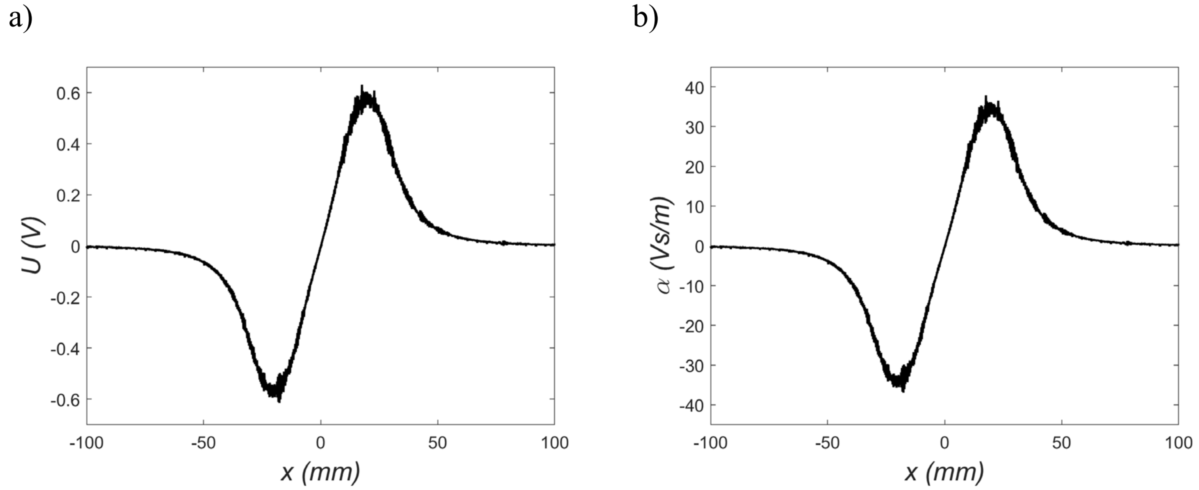

*Fig.1. The measured curve U=f(x) (a) and approximation α=f(x) (b) based on it.*

The trend $\alpha=f(x)$ can be described by a polynomial [6]:

$$\alpha(x)=\alpha_0+\alpha_1 x+\alpha_2 x^2+\alpha_3 x^3+\ldots+\alpha_n x^n. \tag{2}$$

Polynomial coefficients were found using the polyfit MATLAB function [7]. Quality analysis of the experimental data fitting by $n^{\text{th}}$ order polynomial was also performed. The fit quality was assessed using the mean absolute error MAE [8]:

$$\Delta = \frac{1}{m}\sum_{i=1}^{m}\left|\alpha_{i,\mathrm{exp}eriment} - \alpha_{i,polynomial}\right|, \tag{3}$$

where: $m$ is points number, $\alpha_{i,experiment}$ is $i^{\mathrm{th}}$ value of electromechanical coupling coefficient calculated from the experiment (1) and $\alpha_{i,polynomial}$ is $i^{\mathrm{th}}$ value of $\alpha$ taken from polyfit approximation (2). The influence of order number - $n$ on the fit error is illustrated in Fig.2a. When $n$>24 it can be assumed that the error $\Delta$ does not change and it is minimal, about 0.32 Vs/m.

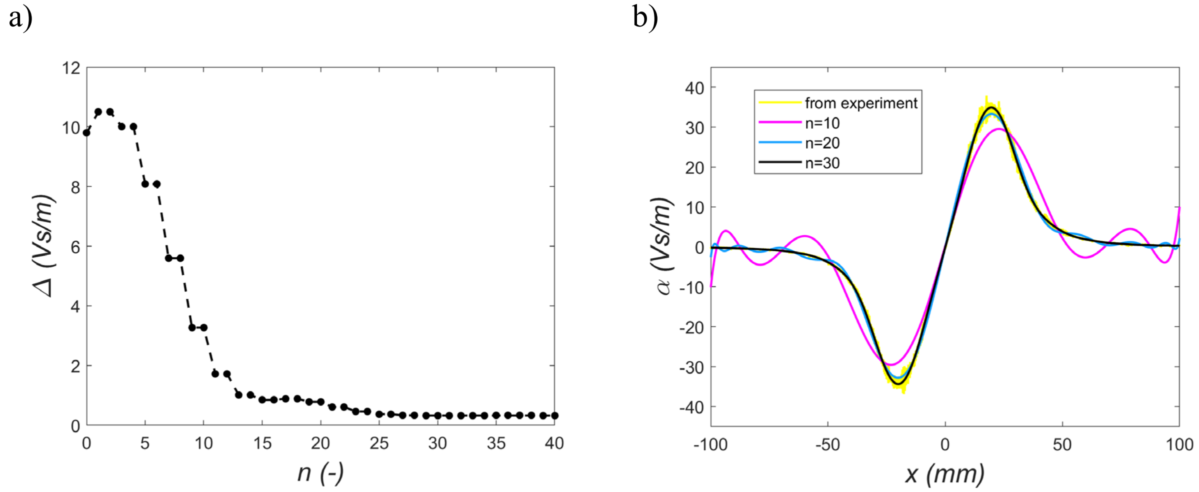

*Fig.2. The fit error Δ=f(n) (a) and selected polynomial approximation α=f(x) (b).*

After analyzing the curves in Fig.2b, it can be concluded that a good representation of the experimental data requires the use of a high-order polynomial.

## Experimental research with double magnet

In the previous section, the electromechanical coupling coefficient model (2) for interaction between a single magnet and inductive coil is given. If this relationship will be applied to a system with two levitating magnets, then the relationship independence between each magnet and the coil should be checked. In this section, this interactions independence is confirmed. To research, two identical magnets were used. The applied orientation of their poles would repeal magnets (see Fig. 3). So, both magnets were connected with a connector. In this situation, the distance between both magnets is constant. This is also important from an analysis point of view. Now, it is much simpler and unambiguous. The structure of the so-called double magnet (magnet - connector - magnet) is shown in Fig.3. This scheme presents and defines the basic elements (1, 2, 3, 4, 5), centers of connector and coil (points O and C, respectively), lengths of magnet and connector ($l$ - magnet, $L$- connector), and coordinate $x$. The experimental tests were repeated for different lengths of the connector $L$: 0 mm, 15mm, 35mm, 55 mm. In this section, the obtained curves for the double magnet and the sum of two characteristics from Fig.1a were compared. This analysis is very important because it can present the possibility of separate consideration of interactions between each magnet and coil. If the superposition principle is confirmed then each interaction magnet-coil can be described by a separate mathematical relationship. The sum of two curves for a single magnet $U=f(x)$ (Fig.1a) must take into account their respective shift:

$$U_1=f(x_1) \qquad where \qquad x_1=x-L/2-l/2, \tag{4}$$

$$U_2=-f(x_2) \qquad where \qquad x_2=x+L/2+l/2, \tag{5}$$

where: $U_1$ is curve $U=f(x)$ taken from Fig.1a and shifted by distance $-L/2-l/2$ in coordinate $x$ domain. Whereas $U_2$ is obtained from the same curve $U=f(x)$, it is inverted and shifted by distance

$L/2+l/2$ in coordinate $x$ domain. This curve reversal is due to the orientation of magnet poles relative to the coil (NS or SN). Now, for the same motion direction and the same position relative coil, the considered separately single magnets will generate voltage with the opposite sign. Finally, the coil voltage from the superposition of both interactions can be written as:

$$U=U_1+U_2. \tag{6}$$

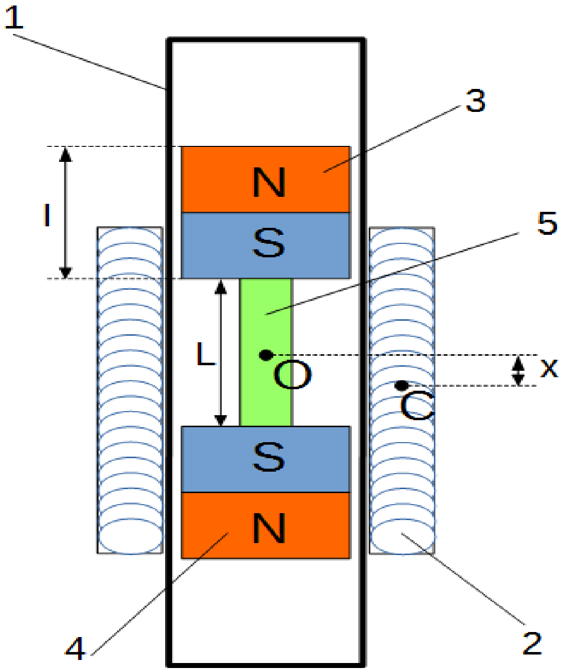

*Fig.3. Scheme of experimental setup.*

*1 - tube, 2 - inductive coil, 3 - top magnet, 4 - bottom magnet, 5 - connector, C - coil center, O - connector center, L - connector length, l - magnet length, x- distance between coil center and double magnet center*

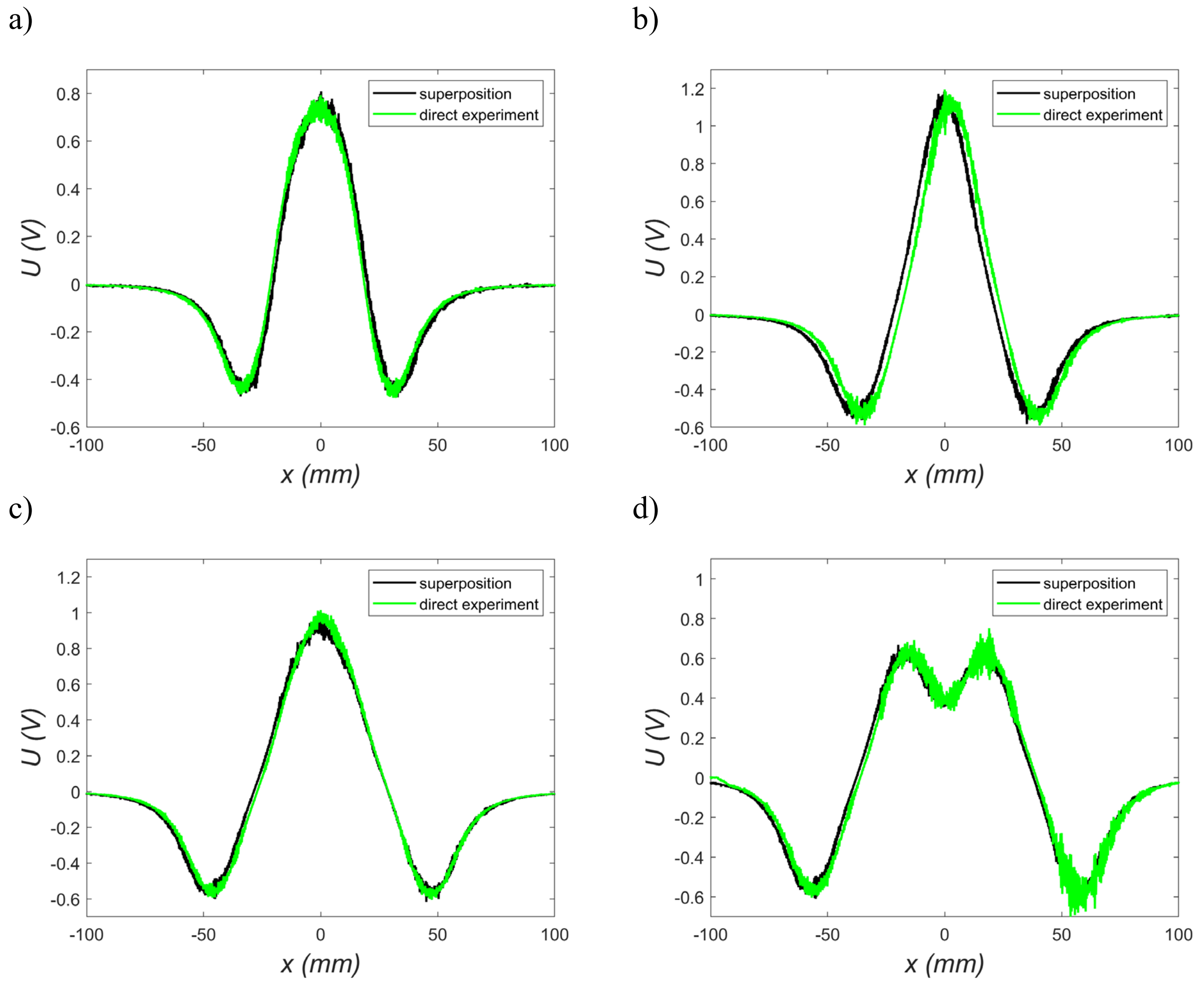

*Fig. 4. Comparison of coil voltage from direct experiment for double magnet and superposition estimation (6).*

*a) L=0 mm, b) L=15 mm, c) L=35mm, d) L=55 mm.*

The obtained results (Fig.4) show that the voltage measured for a system with a double magnet can be mapped using the superposition principle. Comparison of both cases (black and green series) have high compatibility. The maximum and minimum values are very similar. Only a very small shift of both signals can be observed. Based on this analysis, it can be concluded that a separate description of the magnet-coil interaction can be used in the modelling process. Finally, the mathematical description of coil voltage for a case with a double magnet can be written in the following form:

$$U=(\alpha_0+\alpha_1 x_1+\alpha_2 x_1^2+\alpha_3 x_1^3+\ldots+\alpha_n x_1^n)v\pm(\alpha_0+\alpha_1 x_2+\alpha_2 x_2^2+\alpha_3 x_2^3+\ldots+\alpha_n x_2^n)v, \qquad (7)$$

where $v$- double magnet speed and sign ± depends on magnet pole orientation: minus - repulsive magnets presented in this paper or plus - attracting magnets.

## Summary

This paper is an important link between previous research [5] and the new concept. In the future, the system with two levitating magnets and one inductive coil will be considered. The presented results give some information, on how to create a new model. The curve of the electromechanical coupling coefficient is strongly nonlinear. It can be described by a polynomial function. However, the polynomial order should be high ($n$>24). The high-order polynomial can generate some problems when the analytical solution will be searched. The polynomial can be replaced by a low-order trigonometric function [9], but this approach can generate even more problems during the analytical solution calculations.

In the mathematical description of the interaction between the top or bottom magnet and the coil can be investigated separately. It results from the analysis of the correctness of the superposition principle. This fact is important. If the superposition principle would not correct, then the model development of a system with two levitating magnets (case without connector) will be very difficult or impossible.

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