

Multiscale failure analysis of fiber-reinforced composite structures via a hybrid cohesive/volumetric nonlinear homogenization strategy

Daniele Gaetano^{1, a}, Fabrizio Greco^{1, b *}, Lorenzo Leonetti^{1, c},
Paolo Nevone Blasi^{1, d} and Arturo Pascuzzo^{1, e}

¹Department of Civil Engineering, University of Calabria, Rende, Italy

^adaniele.gaetano@unical.it, ^bfabrizio.greco@unical.it, ^clorenzo.leonetti@unical.it,
^dpaolo.nevoneblasi@unical.it, ^earturo.pascuzzo@unical.it

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Abstract. In this work, a novel multiscale model for softening periodic microstructures is proposed, relying on a nonlinear homogenization method combined with a cohesive/volumetric finite element model. This strategy is able to overcome the mesh sensitivity issues usually experienced by purely volumetric homogenization techniques in presence of strain localization. As the main ingredient of the proposed approach, a microscopically informed traction-separation law for the embedded interfaces is extracted, starting from the homogenized bulk behavior obtained for a suitably chosen Repeating Unit Cell (RUC) subjected to different macro-strain paths. The present approach has been fully validated by performing several numerical simulations of the main damage phenomena experienced by fiber-reinforced composite structures, with special reference to transverse micro-cracking. Finally, to investigate the reliability and the accuracy of the proposed model, a comparison with direct simulations performed on fully meshed specimens has been presented, in terms of both load-displacement curves and associated crack patterns.

Introduction

Composite materials are nowadays being frequently used in a wide variety of engineering fields due to their superior mechanical properties compared to conventional single-phase materials. Their exceptional capability to face extreme loads is essentially due to their peculiar microstructural arrangements. Nevertheless, such engineered materials are prone to distinctive failure modes, such as delamination, matrix cracking, fiber/matrix debonding, fiber breakage and buckling [1].

Fully microscopic models are usually required for accurately simulating all these failure mechanisms, although being unpractical for large-scale problems due to the huge computational cost [2]. As a consequence, more effective strategies have been widely used in the technical literature. Most of these strategies are based on micromechanical and/or multiscale models [3-7].

However, traditional micromechanical approaches are not appropriate when strain localization is likely to occur. This is essentially due to the ill-posedness of the resulting macroscopic boundary value problem. To overcome such a drawback, more advanced approaches have been proposed, including higher-order [8], coupled-volume [9], micropolar [10,11], and continuous-discontinuous [12-16] homogenization. Most of these approaches have been used in the spirit of the FE² methods [17], which are often too costly, especially if applied to real-life structural applications.

In this work, a novel continuous/discontinuous multiscale model for periodic microstructures is proposed, based on a hybrid cohesive/volumetric hierarchical homogenization, in which the micro- and macro-scales are only one-way coupled. In particular, two independent homogenized constitutive responses are extracted: (i) a homogenized anisotropic damage evolution law, valid up to the occurrence of strain localization; and (ii) a homogenized mixed-mode traction-separation law. The main advantage of the present multiscale approach relies in the possibility to derive the overall mechanical response in a very efficient manner, by performing off-line computations.

The present approach is validated by performing failure analyses of fiber-reinforced composite beams experiencing transverse micro-cracking, and by comparing the related numerical results with those arising from direct simulations performed on fully meshed specimens.

Theoretical formulation of the cohesive/volumetric homogenization approach

A discretized 2D macroscopic solid Ω_M is considered here (the subscript M denoting the macro-scale), subjected to external tractions $\bar{\mathbf{t}}_M$ and imposed displacements $\bar{\mathbf{u}}_M$ on its Neumann and Dirichlet boundaries, respectively (see Fig. 1a). This solid is damageable, so that fracture is represented via a Diffuse Interface Modeling (DIM) approach, according to which cohesive interfaces, denoted by Γ_M^{coh} , are placed along all the internal mesh boundaries (for additional details see, for instance, [18-23]). Given the periodic nature of the considered microstructure, a suitable Repeating Unit Cell (RUC) is defined to microscopically derive the bulk and interface constitutive relations, which are valid before and after strain localization, respectively (see Figs. 1b and 1c).

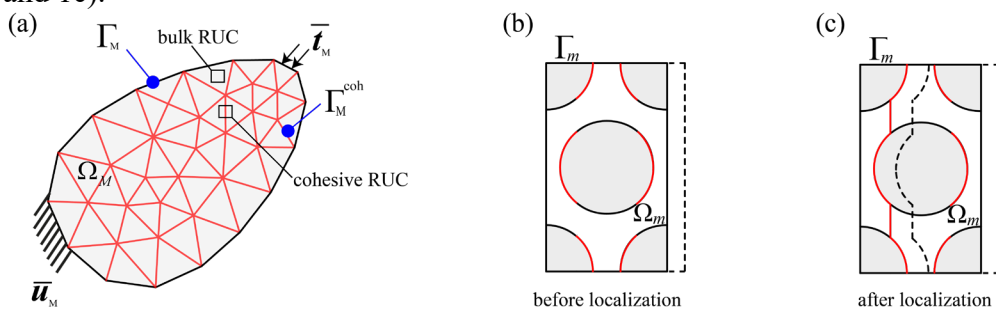


Fig. 1. Schematic of the cohesive/volumetric homogenization approach: (a) homogenized macroscopic problem; (b) bulk microscopic problem; (c) cohesive microscopic problem.

Under the assumptions of scale separation and local periodicity, the macro-stress and strain fields, indicated by σ_M and ϵ_M respectively, are defined in a standard manner as functions of boundary tractions \mathbf{t}_m and displacements \mathbf{u}_m :

$$\sigma_M = \frac{1}{|\Omega_m|} \int_{\Gamma_m} \mathbf{t}_m \otimes \mathbf{x}_m dS, \quad \epsilon_M = \frac{1}{|\Omega_m|} \int_{\Gamma_m} \mathbf{u}_m \otimes_s \mathbf{n}_m dS. \tag{1}$$

In Eq. (1), Ω_m and Γ_m represent the RUC and its external boundary, respectively, whereas \mathbf{x}_m is a generic material point inside the RUC, and \mathbf{n}_m is the outer unit normal at the RUC boundary. The micro-to-macro transition based on the use of Eq. (1) is no longer objective if applied after strain localization has appeared. The resulting localization band, whose width goes to zero in a cohesive model, is described as a zero-thickness interface, equipped with a homogenized traction-separation law, which can be extracted from the homogenized bulk response by using the following relations:

$$\mathbf{t}_M^{\text{coh}} = \sigma_M \mathbf{n}_M, \quad \frac{1}{h} [[\mathbf{u}_M]] \otimes_s \mathbf{n}_M = \epsilon_M. \tag{2}$$

\mathbf{n}_M being the normal to the macro-crack (supposed to be a priori known coherently with the Diffuse Interface Model adopted at the macro-scale), and h is the RUC size measured parallel to \mathbf{n}_M . The homogenized traction-separation law, as computed via Eq. (2), is subsequently depurated from the hardening contribute, associated with the early crack propagation prior to strain localization.

Numerical implementation of the proposed multiscale model

The proposed hybrid cohesive/volumetric nonlinear homogenization has been numerically implemented by considering three operational steps:

1. Computation of the undamaged elasticity tensor by means of the solution of linear boundary value problems for the RUC subjected to different pure macro-strain paths.
2. Computation of the homogenized damage evolution law, extracted from the stress-strain relations obtained via a nonlinear bulk homogenization along different loading paths.
3. Computation of the homogenized traction-separation law, obtained by projecting the previous nonlinear stress-strain relations along \mathbf{n}_M (which varies depending on the actual orientation of the embedded cohesive interfaces for the macro-scale problem).

Numerical experiments: three-point bending test on a fiber-reinforced beam

The case study considered here for validation purposes is the three-point bending test performed on a fiber-reinforced composite beam, analyzed by Canal et al. [24]. Here, plane strain is assumed. The composite beam, whose geometry is depicted in Fig. 2a (with thickness of 2 mm), possesses a periodic microstructure, made with an epoxy matrix and glass fibers, having diameter of 15 μm and volume fraction of 54%, placed in a hexagonal arrangement. The elastic parameters (Young’s modulus and Poisson’s ratio) of the two constituents are listed in Table 1. Matrix cracking is described by an isotropic damage model with linear softening, having a tensile strength of 75 MPa and a fracture energy of 200 N/m, whereas fiber/matrix debonding is described by a mixed-mode cohesive zone model with linear softening, whose parameters are reported in Table 2.

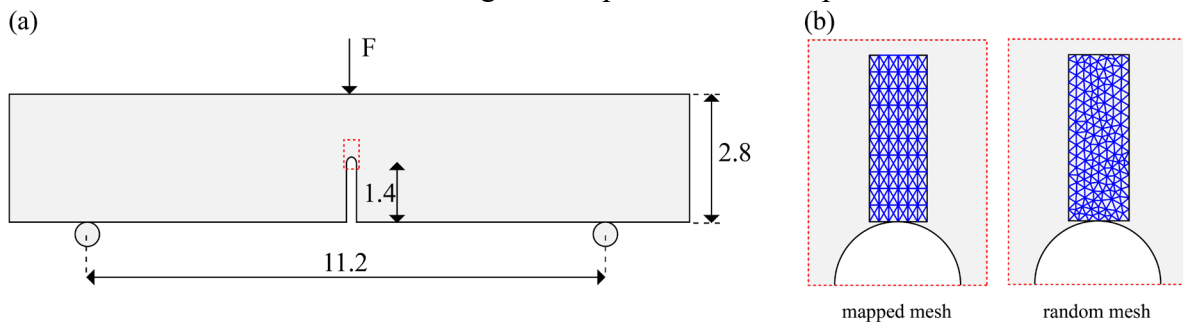


Fig. 2. Three-point bending test configuration: (a) geometry and boundary conditions of the macroscopic specimen (dimensions in mm); (b) detail of the mesh configurations within the damageable zone.

Table 1. Elastic parameters of the constituents.

Component	E [GPa]	ν [-]
Matrix	3.5	0.35
Fiber	74.0	0.20

Table 2. Elastic and inelastic parameters of the fiber/matrix interfaces.

$K_n = K_s$ [N/mm ³]	σ_{nc} [MPa]	σ_{sc} [MPa]	G_{Ic} [N/m]	G_{IIc} [N/m]
1.00×10^8	50.0	75.0	150	150

The proposed multiscale model has been used to derive the nonlinear structural behavior of this composite beam. According to the steps reported in the previous section, the undamaged elasticity tensor, as well as the homogenized damage evolution and traction-separation laws for the given microstructure have been computed. Then, a macro-scale analysis of the beam, here referred to as Multiscale Numerical Simulation, is performed by using the previous databases as material inputs. The macro-scale response is reported in terms of the applied force, F , as a function of the Crack Mouth Opening Displacement, CMOD. With the aim of investigating the influence of the mesh topology, two discretizations have been considered, i.e., a cross-triangle mapped and a Delaunay (random) mesh, shown in Fig. 2b. For validation purposes, the results of these multiscale analyses, or Multiscale Numerical Simulations (MNSs), have been compared with those obtained via a

Direct Numerical Simulation (DNS), in which the microstructural details are explicitly modeled. As shown in Fig. 3a, the MNS results obtained with the mapped mesh are almost coincident with the (reference) DNS results. Conversely, a little overestimation (of about 3%) of the peak load is found for the unstructured mesh (see Fig. 3b) This is not due to the homogenization scheme, but rather to the artificial crack path tortuosity induced by the DIM approach used at the macro-scale.

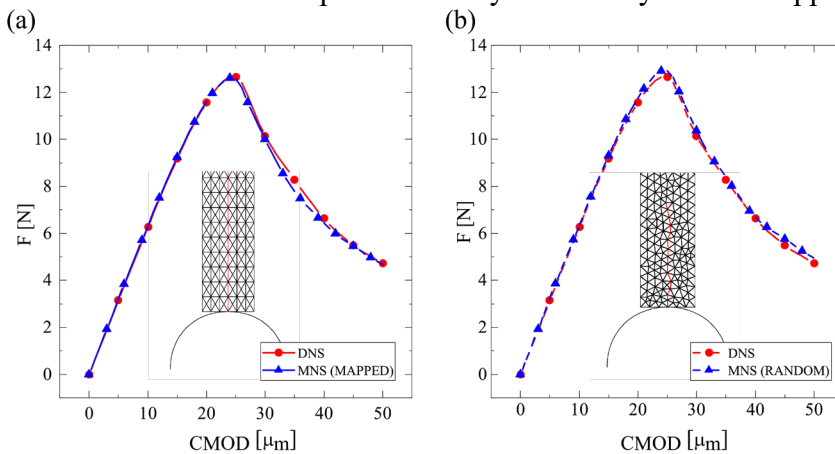


Fig. 3. Comparison between direct and multiscale simulations by using mapped (a) and random (b) meshes.

Conclusions

In this work, a novel hybrid cohesive/volumetric multiscale model is presented, to be adopted for the accurate failure simulation in composite structures. Such a model is based on a diffuse interface approach used in synergy with a continuous/discontinuous nonlinear homogenization scheme. Its main advantage is its greater efficiency with respect to most of the existing multiscale approaches, being related to derivation of homogenized bulk and interface responses via off-line computations.

As a validation step, the present model has been applied for analyzing the failure of a fiber-reinforced beam experiencing transverse cracking under Mode-I fracture conditions. Furthermore, the comparison with a fully detailed model has demonstrated that the proposed numerical strategy preserves a high accuracy, in terms of both load-carrying capacity and average cracking pattern.

A possible improvement of the present model could be the incorporation of plasticity [25] as well as the adoption of more efficient crack models based on moving mesh approaches [26-28].

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