

Nonlinear Analysis of Beams made of High Strength Concrete Prestressed with Unbonded Tendons

KOWALSKI Damian^{1,a*}

¹ Czestochowa University of Technology, Faculty of Civil Engineering, Department of Construction Processes Engineering, Dabrowskiego 69, 42-201 Czestochowa, Poland

^adamian.kowalski@pcz.pl

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Abstract. The paper presents numerical analysis methods of High-Strength Concrete (HSC) beams prestressed with unbonded tendons. Furthermore, it compares obtained results with experimental data from the literature. Prestressing tendons have been modelled in a discrete form, using one-dimensional finite elements. A temperature drop inflicted prestress force. Contact issues have been considered, i.e. friction and pressure at the interface between the cable and the duct wall. In the work, it was found that it is possible to obtain satisfactory accuracy of results with the model in use. Accurate P- Δ (load-deflection) curves were achieved matching experimental data.

Introduction

Main objective of this research was to test the possibility and the feasibility of nonlinear concrete modelling for beams made of High Strength Concrete (HSC), prestressed with unbonded tendons. A damage-plasticity model of concrete and a bilinear elastoplastic model for prestressing tendons were utilized. Both these models proved their accuracy for reinforced concrete structural elements modelling, yet there are no tests of these models, known to the author, performed for prestressed structures.

The experimental data have been taken from [1]. A scheme of used test setup is shown in Fig. 1. The tested beams were subjected to 4-point bending. This allows obtaining a section free from transversal forces due to external load.

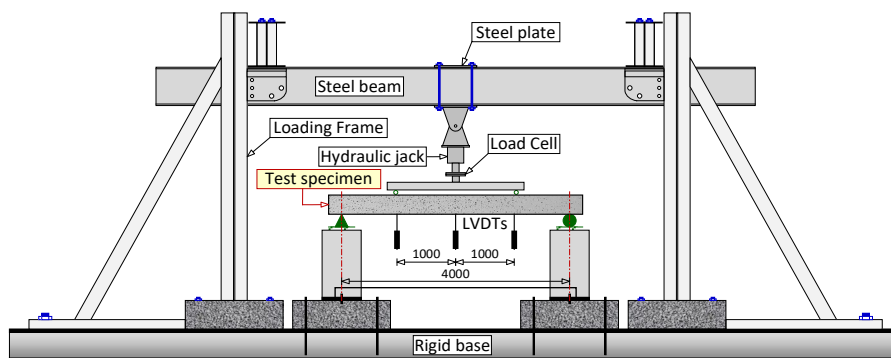


Fig. 1. Scheme of test set-up of beams.

From a variety of beams presented in [1], two of them have been chosen (namely B8 and B9 in original paper). The results were then compared with the results for the respective beams with bonded tendons (B2 and B5 in source paper). The results for bonded tendons were invoked from [2]. All the beams were partially prestressed and contained, except for the tendon, also passive

reinforcement. The difference between pair of beams B2&B8 and the pair B5&B9 was a concrete class. Compressive strength of the former pair is ~73 MPa while the latter pair was made of ~96 MPa concrete. Basic geometrical data of investigated beams are presented in Fig. 2.

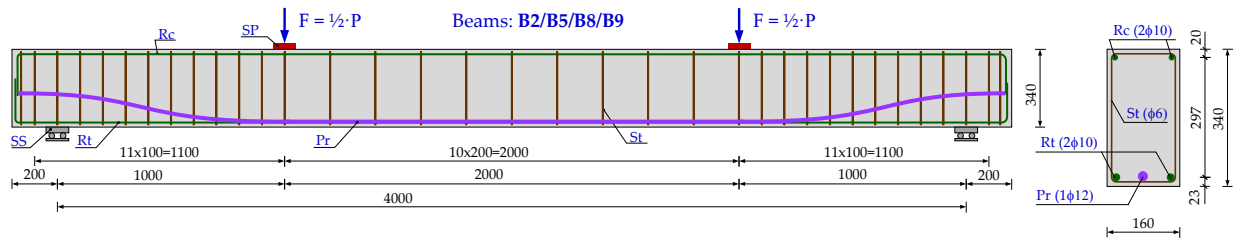


Fig. 2. Scheme of analyzed beams.

The values of concrete parameters not mentioned in the respective article were calculated based on [3]. The biaxial compressive strength of concrete was determined concerning uniaxial strength, according to [4]. The essential factors were gathered in Table 1.

Table 1. Basic concrete parameters

Sample	Young's modulus, E_{cm} , [GPa]	Uniaxial compressive strength, f_{uc} , [MPa]	Biaxial compressive strength, f_{bc} , [MPa]	Poisson's ratio, ν	Intersection point abscissa between compression cap and Drucker-Prager yield function, σ_V^C , [MPa]
B2	40.27	75	86.25	0.2	-57.5
B5	43.50	97	111.55	0.2	-74.4
B8	39.78	72	82.8	0.2	-55.2
B9	43.23	95	109.25	0.2	-72.8

Methods

Finite element model. Both concrete matrix and steel plates were modelled using 8-node hexahedron (brick) finite elements with 3 DOFs (degrees of freedom) in each node. Due to symmetry, only half of the beam has been modelled (see Fig.3). Based on Timoshenko's theory, the prestressing tendons were modeled like sets of 1-dimensional 2-node beam elements. In the case of unbonded tendons, a simplified rectangle-shaped channel was assumed (see Fig.4). This is due to microplane formulation requirement for regular-shaped finite elements of the concrete matrix. Passive reinforcement has been modelled using simple 1-dimensional 2-node link elements having 1 DOF at each node, associated with longitudinal stiffness.

Prestress force in the cables was applied to the tendon in the form of a temperature drop, according to Eq. 1.

$$\Delta t = - \frac{F}{\alpha EA} \tag{1}$$

where:

Δt – temperature drop in the cables during prestressing,
 F – prestressing force,
 α – thermal expansion coefficient,
 E – Young's modulus,
 A – cable cross-sectional area.

Prestress force has been taken as 0.9 of the values given in the paper [1]. It was established based on many trial-and-error simulations on models created for all the prestressed beams described in this research.

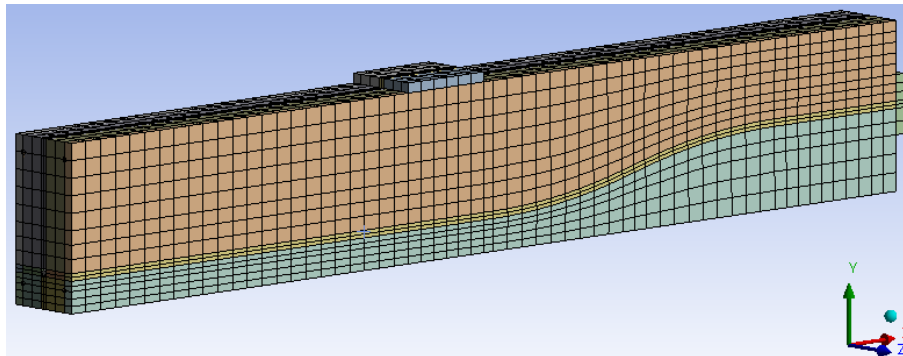


Fig. 3. Finite element model of a sample beam.

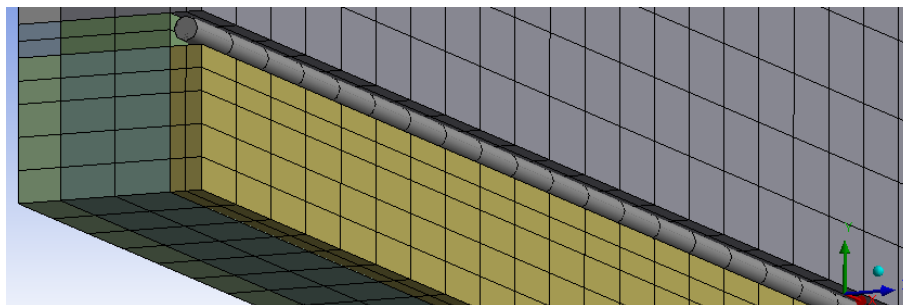


Fig. 4. Unbonded tendon model in the square channel.

Concrete model. A state-of-the-art concrete model was utilized and implemented in ANSYS Mechanical software; namely, the microplane coupled damage-plasticity (widely described in many papers: before the year 2000 [5-9], after the year 2000 [10-17] and the last five years [18-24]) with enhanced gradient regularization. The regularization uses 2 additional degrees of freedom per node, though it reduces solution sensitivity to finite element mesh size. Parameters for the model were taken by trial and error simulations and sensitivity analyses.

Reinforcing steel model. A bilinear model of material, elastoplastic, with kinematic hardening, was adopted. The kinematic hardening takes account of the Bauschinger effect. Fig. 5 presents a finite element model of reinforced bars and prestressing tendons.

Contact parameters. As mentioned above, the tendon channel has been modeled as simplified rectangular in transversal cross-section. Contact has been defined as fully nonlinear, with stiffness updates every load step. The friction coefficient between tendon and concrete surface was taken to equal 0.2.

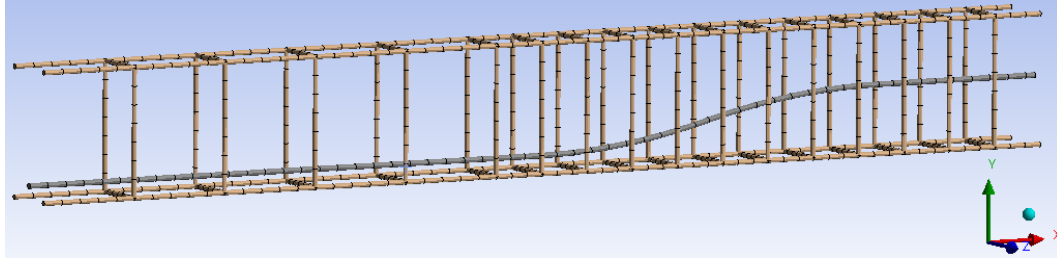


Fig. 5. Finite elements model of active (prestressed) and passive (non-prestressed) steel reinforcement.

Results

Fig. 4 presents load – mid-span displacement curves of all four tested beams. An excellent agreement of the simulation results with experimental data was achieved. This proves the usefulness of the applied model for nonlinear simulations of prestressed concrete structures, including HSC. A typical map of plastic strains is depicted in Fig. 6. It reflects cracks distribution over the concrete matrix.

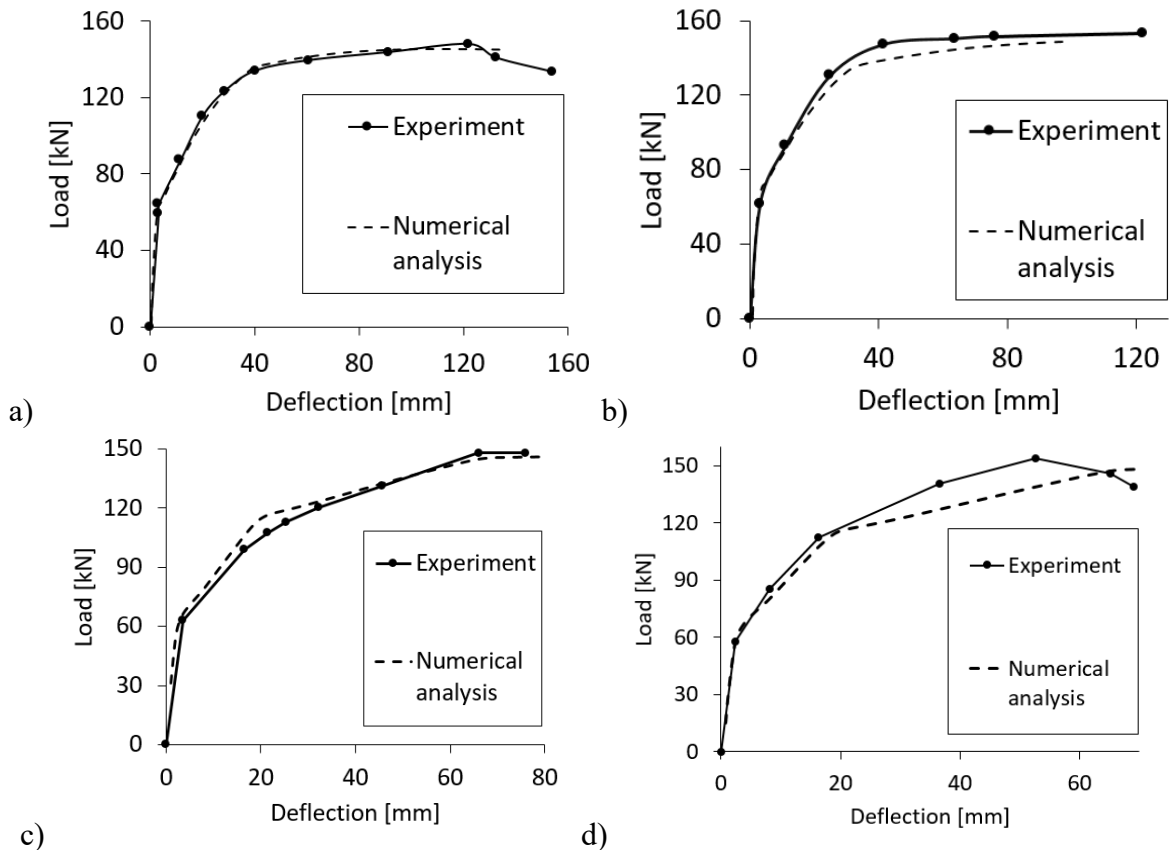


Fig. 6. Comparison of numerical and experimental load-deflection curves of beams B2 (a), B5 (b), B8 (c) and B9 (d).

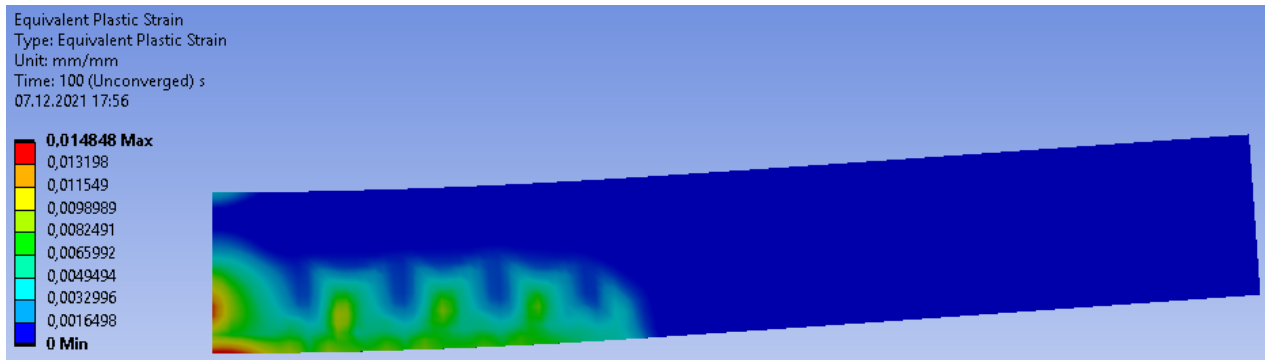


Fig. 7. Typical cracks distribution in tested beam (expressed by means of plastic strains).

Summary

The presented analysis is, of course, critical primarily in the area of concrete applications [25, 26] but also for all predictive analyzes [27-29], also in other scientific areas like materials science [30-33]. It may also creatively influence production engineering [34] when analyzing heavy-duty materials exposed to many factors, including exposure to corrosion [35-37], bio-corrosion in biotechnology [38-40] and agriculture [41-43], or high loads in superheaters [44, 45] and turbines [46, 47].

The paper presents the results of a nonlinear analysis of beams made of High Strength Concrete (HSC), prestressed with unbonded tendons and compares them to the available experimental and numerical data. The comparison shows that the curves match very well for bonded tendons, where there are no contact issues involved. On the other hand, the differences become slightly more significant when unbonded tendons are engaged. A possible reason for this might be modeling the tendons with 1-dimensional finite elements and associated with its contact formulation. Nevertheless, the accuracy, in this case, is still satisfying. However, further research is needed directed towards more accurate tendon modeling.

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