Spectrum Analysis of Mesons using Nikiforov-Uvarov Functional Analysis Method

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Abstract. The analytical expressions for the energy eigenvalues and eigenfunctions are computed for Coulomb perturbed potential by solving the Schrodinger equation within the framework of the Nikiforov-Uvarov functional analysis method and applying the Greene-Aldrich approximation. Using the energy eigenvalue expression, we have determined the mass spectra of $c\bar{s}$ and $\bar{b}c$ mesons. The results of present study are in good agreement with experimental, relativistic and other relevant works available in the literature.

Introduction

As we know that most of the quantum mechanical systems can be studied by determining the bound state solutions to Schrodinger equation (SE) within a suitable potential model. Thereby, within the framework of an appropriate potential model and a proper mathematical approach, one can understand the behaviors of quantum mechanical systems. Such theoretical models also help to investigate the complicated structures, the mass spectra and radiative transitions width of various mesons. In non-relativistic quantum mechanics, experimentally observed vital physical properties can be reproduced with a suitable interaction potential model which, in general, is a function of relative separation between quark and antiquark pairs. In literature, there exist several important studies related to computation of mass spectra of mesons; a brief of those is as follows.

Inyang et. al. [1] studied a temperature dependent Yukawa potential by replacing the screening parameter with the Debye mass and obtained mass spectra of heavy mesons by solving the SE with series expansion method. Moazami et. al [2] solved the SE with the Cornell potential and calculated the mass spectra of some mesons. Using power series and asymptotic iteration method, Ramesh et. al [3, 4] calculated the mass spectra of charmonium and bottomonium by solving the N-dimensional SE. By considering a generalized interaction potential, Richa et. al [5] solved the SE via asymptotic iteration method and calculated the mass spectra of some heavy and light mesons. Some other important studies related to mass spectra of mesons are available in refs. [6-9].

So, to obtain better results for meson systems, here, we use the Coulomb perturbed potential (CPP) as an interaction potential i.e.

$$V(r) = ar^2 + br - \frac{c}{r}, \tag{1}$$

where a, b and c are potential parameters and should be chosen appropriately such that the obtained results be comparable with available experimental and other theoretical outcomes.

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Formalism

In literature, there are many robust and easy to apply mathematical techniques to solve the SE with exactly or approximately. Such approaches include Nikiforov Uvarov (NU) method [10], Lie algebraic approach [11], asymptotic iteration method (AIM) [12], parametric Nikiforov Uvarov (pNU) method [13], and many others. Here in our present calculations, however we have utilized a relatively new method, the Nikiforov Uvarov functional analysis (NUFA) method [14].

The non-relativistic radial SE for the potential (1) is written as

$$\frac{d^2R(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left(E-ar^2+br+\frac{c}{r}-\frac{l(l+1)\hbar^2}{2\mu r^2}\right)R(r)=0 \tag{2}$$

Now by invoking the Greene-Aldrich approximation [15] for the centrifugal term

$$\frac{1}{r^2}\approx\frac{\alpha^2}{(1-e^{-\alpha r})^2}\ , \tag{3}$$

E q. (2) reduces to

$$\frac{d^2R(r)}{dr^2}+\frac{\alpha^2}{(1-e^{-\alpha r})^2}\left[\frac{2\mu E(1-e^{-\alpha r})^2}{\hbar^2\alpha^2}-\frac{2\mu a(1-e^{-\alpha r})^4}{\hbar^2\alpha^4}-\frac{2\mu b(1-e^{-\alpha r})^3}{\hbar^2\alpha^3}+\frac{2\mu c(1-e^{-\alpha r})}{\hbar^2\alpha}-l(l+1)\right]R(r)=0 \tag{4}$$

By putting $s=e^{-\alpha r}$ in above equation and neglecting higher powers of ‘s’ in the resultant expression, we obtain

$$\frac{d^2R(r)}{dr^2}+\frac{1}{s}\frac{dR(s)}{ds}+\frac{1}{s^2(1-s)^2}\left[\left(\frac{2\mu E}{\hbar^2\alpha^2}-\frac{12\mu E}{\hbar^2\alpha^4}-\frac{6\mu b}{\hbar^2\alpha^3}\right)s^2+\left(\frac{8\mu a}{\hbar^2\alpha^2}-\frac{4\mu E}{\hbar^2\alpha^2}+\frac{6\mu b}{\hbar^2\alpha^3}-\frac{2\mu c}{\hbar^2\alpha}\right)s+\left(\frac{2\mu E}{\hbar^2\alpha^2}-\frac{2\mu a}{\hbar^2\alpha^4}-\frac{2\mu b}{\hbar^2\alpha^3}+\frac{2\mu c}{\hbar^2\alpha}-l(l+1)\right)\right]R(s)=0\,. \tag{5}$$

Eq. (5) can further be written as

$$\frac{d^2R(r)}{dr^2}+\frac{1}{s}\frac{dR(s)}{ds}+\frac{1}{s^2(1-s)^2}[-\xi_1s^2+\xi_2s-\xi_3]R(s)=0, \tag{6}$$

where,

$$\xi_1=-\frac{2\mu E}{\hbar^2\alpha^2}-\frac{12\mu E}{\hbar^2\alpha^4}-\frac{6\mu b}{\hbar^2\alpha^3}=-\frac{2\mu E}{\hbar^2\alpha^2}+\tau_1\,, \tag{7a}$$

$$\xi_2=-\frac{4\mu E}{\hbar^2\alpha^2}+\frac{6\mu b}{\hbar^2\alpha^3}-\frac{2\mu c}{\hbar^2\alpha}+\frac{8\mu a}{\hbar^2\alpha^2}=-\frac{4\mu E}{\hbar^2\alpha^2}+\tau_2\,, \tag{7b}$$

$$\xi_3=-\frac{2\mu E}{\hbar^2\alpha^2}+\frac{2\mu a}{\hbar^2\alpha^4}+\frac{2\mu b}{\hbar^2\alpha^3}-\frac{2\mu c}{\hbar^2\alpha}+l(l+1)=-\frac{2\mu E}{\hbar^2\alpha^2}+\tau_3\,. \tag{7c}$$

Comparing Eq. (5) with the following standard equation of NUFA method [14]

$$\frac{d^2\psi(s)}{ds^2}+\frac{\alpha_1-\alpha_2s}{s(1-\alpha_3s)}\frac{d\psi(s)}{ds}+\frac{1}{s^2(1-\alpha_3s)^2}[-\xi_1s^2+\xi_2s-\xi_3]\psi(s)=0, \tag{8}$$

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one can obtain

$\alpha_1 = \alpha_2 = \alpha_3 = 1,$

$\mu_1 = \sqrt{\varepsilon + \tau_3}\,,$

$$\nu_1 = \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4(\tau_1 - \tau_2 + \tau_3)}\ . \tag{9}$$

The equation for finding the energy eigenvalue using the NUFA method [14] is given as

$$\mu_1^2 + 2\mu_1\left(\nu_1 + \frac{\alpha_2}{\alpha_3} - 1 + \frac{1}{\sqrt{\alpha_3}}\right) + \left(\nu_1 + \frac{\alpha_2}{\alpha_3} - 1 + \frac{1}{\sqrt{\alpha_3}}\right)^2 - \left(\frac{\alpha_2}{\alpha_3} - 1\right) - \frac{\xi_1}{\alpha_3^2} = 0. \tag{10}$$

Using Eqs. (7a) and (9) in eq. (10), the energy eigenvalue expression for the CPP is written as

$$E = -\frac{\hbar^2\alpha^2}{2\mu}\left[\frac{(-(\nu_1+n)^2+\tau_1-\tau_3)}{2(\nu_1+n)}\right]^2 + \frac{\hbar^2\alpha^2\tau_3}{2\mu}; \tag{11}$$

and the corresponding eigen function is thus written as

$$R_n(r) = N(e^{-\alpha r})^{\sqrt{\varepsilon+\tau_3}}\,(1 - e^{-\alpha r})^{\frac{1}{2}\pm\frac{1}{2}\sqrt{1+4(\tau)}} \times 2F_1(-n, n + 2(\mu_1 + \nu_1), 1 + 2\mu_1; e^{-\alpha r}). \tag{12}$$

Spectrum Analysis of Mesons

So, in order to calculate the mass spectra of mesons, we exploit the following well known equation [4]

$$M = m_q + m_{\bar{q}} + E_{nl}, \tag{13}$$

here m_q and $m_{\bar{q}}$ are the masses of quark and antiquark respectively. Thus, by using Eq. (11) in the above equation, we determine the equation for finding the mass spectra of mesons as

$$M = m_q + m_{\bar{q}} - \frac{\hbar^2\alpha^2}{2\mu}\left[\frac{(-(\nu_1+n)^2+\tau_1-\tau_3)}{2(\nu_1+n)}\right]^2 + \frac{\hbar^2\alpha^2\tau_3}{2\mu}\ . \tag{14}$$

Further, to check the correctness of analytical results, we have calculated mass spectra of $c\bar{s}$ and $\bar{b}c$ mesons by considering m_b =4.823 GeV, m_s =0.419 GeV and m_c =1.209 GeV. The numerical values of the potential parameters are taken from ref. [5].

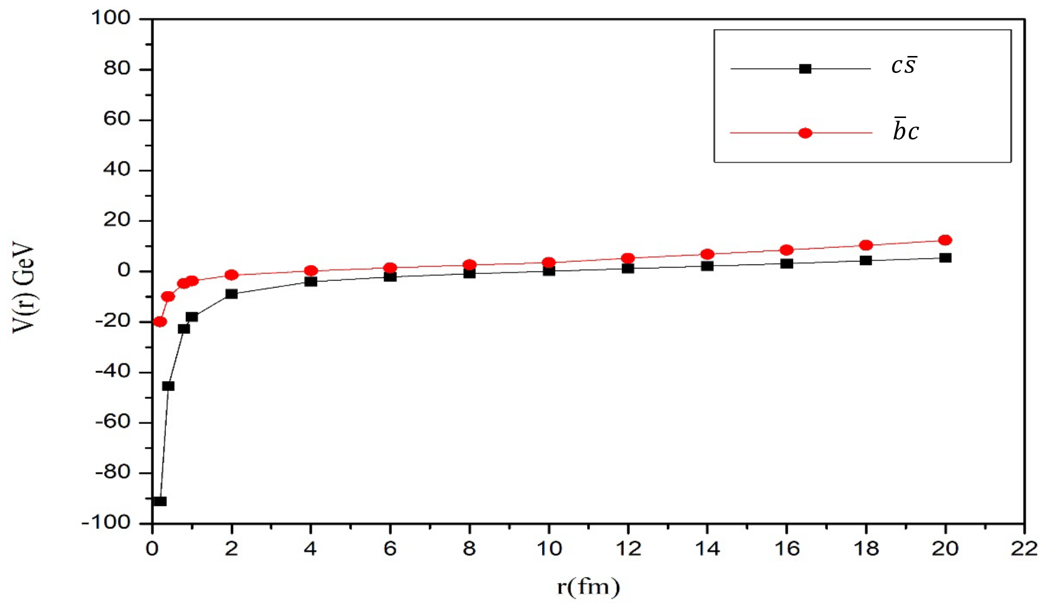

Fig. 1: Behavior of Coulomb perturbed potential with 'r'.

Table 1: *Mass spectra of $c\bar{s}$ in GeV (a = 0.0123 GeV^3, b= 0.0688 GeV^2, c = 18.2252, α=0.0725, μ= 0.3111 GeV)*

State	Present Work	Experimental [9]	Relativistic [6]	Ref. [5]	Ref. [16]
1S	2.517	-	2.129	2.512	1.968
1P	2.548	-	2.549	2.649	2.565
1D	2.610	2.859	2.899	2.859	2.857
2S	2.755	2.709	2.732	2.709	2.709
2P	2.784	-	3.018	2.860	-
3S	2.956	-	3.193	2.906	2.932
4S	3.124	-	3.575	3.102	-

Table 2: *Mass spectra of $\bar{b}c$ in GeV (a = 0.0204 GeV^3, b = 0.2209 GeV^2, c = 4.0087, α=0.2187, μ = 0.9666 GeV)*

State	Present Work	Experimental [8]	Relativistic [7]	Ref. [5]	Ref. [16]
1S	6.277	6.277	6.332	6.277	6.277
1P	6.365	-	6.734	6.340	7.042
1D	6.531	-	7.072	6.452	-
2S	6.460	-	6.881	6.814	7.383
2P	6.532	-	7.126	6.851	6.663
3S	6.558	-	7.235	7.235	7.206
4S	6.591	-	-	7.889	-

Concluding Remarks

Here, we have explored the NUFA method for solving the Schrodinger equation and obtained analytical expressions for the bound state energy eigenvalues and eigenfunctions within the framework of the Coulomb perturbed potential. To check the suitability of the choice of potential and applicability of the NUFA-method, we utilized the energy eigenvalue expression (11) to compute the mass spectra of $c\bar{s}$ and $\bar{b}c$ mesons with suitable values of potential parameters chosen from literature. The obtained results are comparable with the earlier published results of similar studies. The behaviour of the Coulomb perturbed potential with separation 'r' is also plotted in Fig.1

Conflicts of interests

The authors declare no conflict of interest.

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